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Integrate the following with respect to their variable

(1) $3te^{2t}$ (2) $x^2 \sin x$ (3) $\sin 7x \cos 2x$ (4) $(2x - 3x^2)/1-x$

(1) $3te^{2t}\,dt$

let $u = 3t$; $dv = e^{2t}$

$du/dt = 3$; $v = \frac{e^{2t}}{2}$

$\int u\,dv = uv - \int v\,du$

$= 3t \cdot \left(\frac{e^{2t}}{2}\right) - \int \frac{e^{2t}}{2} \cdot 3$

$= \frac{3te^{2t}}{2} - \frac{3}{2}\int e^{2t}$

$= \frac{3te^{2t}}{2} - \frac{3e^{2t}}{4} + C$

(2) $x^2 \sin x\,dx$

let $u = x^2$; $dv = \sin x$

$du/dx = 2x$; $v = -\cos x$

$\int u\,dv = uv - \int v\,du$

$= x^2 \cdot (-\cos x) - \int -\cos x \cdot (2x)$

$= -x^2 \cos x - \int -\cos x\, 2x$

Integrating $-2x\cos x$ by part

let $u = -2x$; $dv = \cos x$

$\frac{du}{dx} = -2$; $v = \sin x$

$\int u\,dv = uv - \int v\,du$

$= -2x\sin x - \int \sin x \cdot (-2)$

$= -2x\sin x + 2\int \sin x$

$= -2x\sin x - 2\cos x + C$

$\therefore \int -2x\cos x = -2x\sin x - 2\cos x + C$

Recall $\int x^2 \sin x\,dx = -x^2\cos x - \int -2x\cos x$

where $\int -2x\cos x = -2x\sin x - 2\cos x + C$

$\therefore \int x^2 \sin x\,dx = -x^2\cos x + 2x\sin x + 2\cos x + C$

③ $\sin 7x \cos 2x\,dx$

Where $\sin A\cos B = \frac{1}{2}[\sin(A+B) + \sin(A-B)]$

$= \frac{1}{2}[(\sin(7x+2x) + \sin(7x-2x)]$

$= \frac{1}{2}[\sin 9x + \sin 5x]$

$\int \sin 7x \cos x\,dx = \frac{1}{2}\int (\sin 9x + \sin 5x)$

Integrating $\sin 9x$; let $u = 9x$

$du/dx = 9$; $dx = du/9$

$\therefore \int \sin u \cdot dx$

$= \int \sin u \cdot du/9$

$= \frac{1}{9}\int \sin u \cdot du$

$= -\frac{1}{9}\cos 9x$

$\therefore \sin 5x = -\frac{1}{5}\cos 5x$

$\therefore \int \sin 7x \cos 2x \, dx$

$$= \frac{1}{2}\left[\frac{-\cos 9x}{9} + \left(\frac{-\cos 5x}{5}\right)\right] + C$$

$$= \frac{-\cos 9x}{18} - \frac{\cos 5x}{10} + C$$

(4) $\int \frac{(2x-3x^2)}{(1-x)}$

$$= -\int \frac{x(3x-2)\,dx}{x-1}$$

let $u = x-1$; $\frac{du}{dx} = 1$; $du = dx$

$x = u+1$; $x^2 = (u+1)^2$

$$= \int \frac{(u+1)(3u+1)}{u}$$

$$= \int \frac{3u^2+4u+1}{u} du$$

$$= \int \left(3u + \frac{1}{u} + 4\right) du$$

$$= 3\int u\,du + \int \frac{1}{u} du + 4\int du$$

$$= 3\left(\frac{u^2}{2}\right) + \ln u + 4u$$

$$= \ln u + \frac{3u^2}{2} + 4u$$

$$= \ln(x-1) + \frac{3(x-1)^2}{2} + 4(x-1)$$

$$\int \frac{(2x-3x^2)}{1-x} = -4(x-1) - \frac{3(x-1)^2}{2} - \ln(x-1).$$

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