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1a-) $y = \sin\left(3/x^2\right)$

$y = \sin 3/x^2$

$y = \sin 3x^{-2}$

let $u = 3x^{-2}$

$\Delta y + y = \sin(u + \Delta u)$

$\Delta y = \sin(u + \Delta u) - y$

$\Delta y = \sin(u + \Delta u) - \sin u$

Recall $\sin A - \sin B = 2\cos \frac{A+B}{2} \sin \frac{A-B}{2}$

$A = u + \Delta u$, $B = u$

$$\frac{A+B}{2} = \frac{u + \Delta u + u}{2} = \frac{2u + \Delta u}{2} = u + \frac{\Delta u}{2}$$

$$\frac{A-B}{2} = \frac{u + \Delta u - u}{2} = \frac{\Delta u}{2}$$

Hence, $\frac{\Delta y}{\Delta u} = 2\cos\left(u + \frac{\Delta u}{2}\right)\sin\left(\frac{\Delta u}{2}\right)$

$\Delta u \times 1/2$

$\sin \lim_{\Delta u \to 0} \cos(u + 0)$ $\lim_{\Delta u \to 0} \left(\frac{\Delta u}{2}\right) / \left(\Delta u / 2\right)$

$\frac{dy}{du} = \cos u \times 1$

$\frac{dy}{dx} = \cos u$

Also $u = 3/x^2$

$\Delta u + u = \frac{3}{(x^2 + \Delta x)^2}$

$\Delta u = \frac{3}{(x + \Delta x)^2} - u$

$\Delta u = \frac{3}{(x + \Delta x)^2} - \frac{3}{x^2}$

$\Delta u = \frac{3x^2 - 3(x + \Delta x)^2}{x^2 (x + \Delta x)^2}$

$\Delta u = \frac{3x^2 - 3(x^2 + 2x\Delta x + (\Delta x)^2)}{x^2 (x + \Delta x)^2}$

$\Delta u = \frac{-6x\Delta x - 3(\Delta x)^2}{x^2 (x + \Delta x)^2}$

$$\frac{\Delta u}{\Delta x} = \frac{-6x\,\cancel{\Delta x}}{x^2(x+\Delta x)^2} \times \frac{1}{\cancel{\Delta x}} - \frac{3(\Delta x)^2}{x^2+(x+\Delta x)^2} \times \frac{1}{\cancel{\Delta x}}$$

$$= \frac{-6x}{x^2(x+\Delta x)^2} - \frac{3\Delta x}{x^2(x+\Delta x^2}$$

$$\lim_{\Delta x \to 0} \frac{\Delta u}{\Delta x} = \frac{-6x}{x^2(x+0)^2} - \frac{3(0)}{x^2(x+0)^2}$$

$$\frac{du}{dx} : \frac{-6x}{x^4} - 0$$

$$\frac{du}{dx} = -6x^3$$

$$\frac{dy}{dx} = \frac{du}{dx} \times \frac{dy}{du}$$

$$= -6x^{-3} \times \cos u$$

recall $u = 3/x^2$

$$\frac{dy}{dx} = \frac{-6}{x^3} \times \cos\left(3/x^2\right)$$

$$= \frac{-6}{x^3} \cos 3/x^2$$

1b.) $y = 4/x^3$

$$y + \Delta y = \frac{4}{(\Delta x + x)^3}$$

$$\Delta y = \frac{4}{(\Delta x + x)^3} - y$$

$$\Delta y = \frac{4}{(\Delta x + x)^3} - \frac{4}{x^3}$$

$$\Delta y = \frac{4x^3 - 4(\Delta x + x)^3}{x^3(\Delta x + \cancel{x})^3}$$

$$\Delta y = \frac{4x^3 - 4[x^3 + 3x^2(\Delta x) + 3x(\Delta x)^2 + (\Delta x)^3]}{x^3[x^3 + 3x^2(\Delta x) + 3x(\Delta x)^2 + (\Delta x)^3]}$$

$$\Delta y = \frac{\cancel{4x^3} - \cancel{4x^3} - 12x^2(\Delta x) - 12x(\Delta x)^2 - 4(\Delta x)^3}{x^3(\Delta x + x)^3}$$

$$\Delta y/\Delta x = \frac{-12x^2(\cancel{\Delta x})}{\cancel{x}^3(\Delta x + x)^3} \times \frac{1}{\cancel{\Delta x}} \quad \frac{-12x^2(\Delta x)}{x^3(\Delta x + x)^3} \times \frac{1}{\cancel{\Delta x}} \quad \frac{-4(\Delta x)^{\cancel{3}\,2}}{x^3(\Delta x + x)^3} \times 1/\cancel{\Delta x}$$

$$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \frac{-12x^2}{x^3(0+x)^3} - 0 - 0$$

$$\frac{dy}{dx} = \frac{-12x^2}{x^6}$$

$$= -12x^{-4}$$

2(a) $\int \frac{1}{x^2+36} \, dx$

$u = x/6 \longrightarrow \frac{du}{dx} = 1/6$

$dx = 6\,du$

$$= \int \frac{6}{36u^2+36} \, du$$

$$= 1/6 \int \frac{1}{u^2+1} \, du$$

also $\int \frac{1}{u^2+1} \, du = \arctan(u)$

$1/6 \int \frac{1}{u^2+1} \, du = \frac{\arctan u}{6}$

recall $u = \frac{x}{6}$

Thus $\displaystyle\int \frac{1}{x^2+36}\,dx = \frac{\arctan(x/6)}{6} + C$

6) $\displaystyle\int \frac{1}{x^3+13}\,dx$

$u = \dfrac{x}{\sqrt{13}} \longrightarrow \dfrac{du}{dx} = \dfrac{1}{\sqrt{13}}$

$dx = \sqrt{13}\,du$

$= \displaystyle\int \frac{\sqrt{13}}{13u^2+13}\,du$

$= \dfrac{1}{\sqrt{13}} \displaystyle\int \frac{1}{u^2+1}\,du$

also $\displaystyle\int \frac{1}{u^2+1}\,du = \arctan(u)$

$\therefore \dfrac{1}{\sqrt{13}} \displaystyle\int \frac{1}{u^2+1}\,du = \frac{\arctan(u)}{\sqrt{13}}$

recall $u = \dfrac{x}{\sqrt{13}}$

thus $\displaystyle\int \frac{1}{x^2+13}\,dx = \frac{\arctan(x/13)}{\sqrt{13}} + c$