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19/MHS01/002
Medicine and Health sciences
Medicine and Surgery
Serial No: 100
MAT 104 Assignment.

1) $\int \frac{2x}{\sqrt{4x^2-1}}\,dx = \frac{1}{2}\sqrt{4x^2-1+c}$

solution

$\int \frac{2x}{\sqrt{4x^2-1}}\,dx$

Let $u = \sqrt{4x^2-1}$, $P = 4x^2-1$

$\therefore u = \sqrt{P}$

$\frac{dP}{dx} = 8x$ $\qquad \frac{du}{dP} = P^{1/2} = \frac{1}{2}P^{-1/2}$

$= \frac{1}{2} \times \frac{1}{\sqrt{P}}$

$= \frac{1}{2} \times \frac{1}{\sqrt{4x^2-1}}$

$\frac{dy}{dx} = \frac{dP}{dx} \times \frac{du}{dP}$

$= 8x \times \frac{1}{2\sqrt{4x^2-1}}$

$\frac{du}{dx} = \frac{4x}{\sqrt{4x^2-1}}$

$dx = \frac{\sqrt{4x^2-1}\,du}{4x}$

$\int \frac{2x \times \cancel{\sqrt{4x^2-1}}\,du}{\cancel{\sqrt{4x^2-1}} \times 4x}$

$\int \frac{1}{2}\,du$

$\frac{1}{2}\int du = \frac{1}{2}u + c$

$= \frac{1}{2} \times \sqrt{4x^2-1} + c$

2) $\int \frac{\sin^{-1}x}{\sqrt{1-x^2}}\,dx = \frac{(\sin^{-1}x)^2}{2} + c$

solution

$\int \frac{\sin^{-1}x}{\sqrt{1-x^2}}\,dx$

let $u = \sqrt{1-x^2}$ $\qquad P = 1-x^2$

$u = \sqrt{P}$ $\qquad \frac{du}{dP} = \frac{1}{2}P^{-1/2}$ $\qquad \frac{dP}{dx} = -2x$

$\frac{du}{dx} = \frac{dP}{dx} \times \frac{du}{dP}$

$\frac{du}{dx} = -\cancel{2}x \times \frac{1}{\cancel{2}} \times \frac{1}{\sqrt{P}}$

$\frac{du}{dx} = \frac{-x}{\sqrt{P}}$

$\frac{du}{dx} = \frac{-x}{\sqrt{1-x^2}}$

$dx = \frac{-\sqrt{1-x^2}\,du}{x}$

from the question

$\int \frac{\sin^{-1}x}{\sqrt{1-x^2}}\,dx$

$\int \frac{\sin^{-1}x}{\cancel{\sqrt{1-x^2}}} \times -\frac{\cancel{\sqrt{1-x^2}}\,du}{x}$

$-\int \frac{\sin^{-1}x\,du}{x}$

But $u = \sqrt{1-x}$

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$u^2 = 1 - x$

$x = 1 - u^2$

$\therefore \int \frac{\sin^{-1}(1-u^2)\,du}{1-u^2}$

$= \ln(\sin^{-1}x) + C$

3) $\int (\tan x)^6 \sec^2 x\, dx$ Ans $= \frac{(\tan x)^7}{7} + C$

$\left\{\begin{array}{l} u = \tan x \\ du = \sec^2 x\, dx \\ u^6\, du = u^7/7 \end{array}\right\}$

<u>solution</u>

$\int (\tan x)^6 \sec^2 x\, dx$

Let $u = \tan x$

$\frac{du}{dx} = \sec^2 x \quad \therefore \quad dx = \frac{du}{\sec^2 x}$

$\int u^6 \times \cancel{\sec^2 x} \times \frac{du}{\cancel{\sec^2 x}}$

$\int u^6\, du = \frac{u^7}{7} + C$

$= \frac{(\tan x)^7}{7} + C$