PHY 102

Explain with the aid of a diagram how you can produce a negatively charged sphere by method of induction.

Answer

Electric charges can be obtained on an object without touching it, by a process called electrostatic induction.

Consider a negatively charged rubber rod brought near a neutral (uncharged) conducting sphere that is insulated so that there is no conducting path to ground as shown in the diagram below. The repulsive force between the electrons in the rods and those in the sphere causes a redistribution of charges on the sphere so that some electrons move to the side of the sphere farthest away from the rod (fig A). The region of sphere nearest the negatively charged rod has an excess of positive charge because of the migration of electrons away from this location. If a grounded conducting wire is then connected to the sphere, as in (fig. B), some of the electrons leave the sphere and travel to the earth. If the wire to ground is then removed (fig C), the conducting sphere ~~and travel to the earth. If the wire to ground is then removed~~ is left with an excess of induced positive charge.

Finally, when rubber rod is removed from the vicinity of the sphere (fig D), the induced positive charge remains on the ungrounded sphere and becomes uniformly distributed over the surface of the sphere.

Fig A

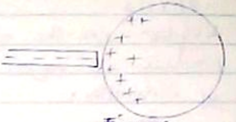


Fig C

Fig. B

Fig. D

1b Each of two small spheres is charged positively, the combined charge being 5.0×10^{-5}C. If each sphere is repelled from the other by a force of 1.0N when the spheres are 2.0m apart, Calculate the charge on each sphere.

Solution making q_1 subject formula

$q_1 + q_2 = 5\times10^{-5}C \quad \therefore q_1 = 5\times10^{-5} - q_2$ Parameters

$F = 1.0N$

$r = 2.0m$

Recall $F = \frac{kq_1q_2}{r^2}$, $1.0N = \frac{9\times10^{9}(q_1)(q_2)}{(2)^2}$

$\therefore 1.0 = \frac{9\times10^{9}(q_1)(q_2)}{4}$

$= 4 = 9\times10^{9}(q_1)q_2$ Replacing $q_1 = 5\times10^{-5} - q_2$

$4 = 9\times10^{9}(5\times10^{-5} - q_2)q_2$

$\therefore 4 = 4.5\times10^{-5}q_2 - 9\times10^{9}q_2^2$

$-9\times10^{9}q_2^2 + 4.5\times10^{-5}q_2 - 4 = 0$

using quadratic formula, $x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$a = 9\times10^{9}, b = 4.5\times10^{-5}, c = -4$

$$q_2 = \frac{-4.5\times10^{-5} \pm \sqrt{(4.5\times10^{-5})^2 - 4(-9.0\times10^9)-4}}{2(-9.0\times10^9)}$$

$$= \frac{-4.5\times10^{-5} \pm \sqrt{(2.025\times10^{-9}) - 4(-9\times10^9)(-4)}}{-18.0\times10^9}$$

$$= \frac{-4.5\times10^{-5} \pm \sqrt{3.6\times10^{10}}}{-18.0\times10^9}$$

$$= \frac{-4.5\times10^{-5} \pm 189736.6596}{-18.0\times10^9}$$

$$q_2 = \frac{-4.5\times10^{-5} - 189736.6596}{-18.0\times10^9}$$

$q_1 = 1.10\times10^{-5}C$

$\therefore$ Recall $q_1 = 5\times10^{-5} - q_2$

making q_2 subject formula

$\therefore$ $q_2 = 5\times10^{-5} - q_1$

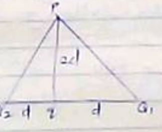
$q_2 = 5\times10^{-5} - 1.10\times10^{-5}$

$q_2 = 3.9\times10^{-5}C$

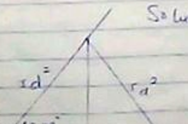
$\therefore$ $q_2 = 3.9\times10^{-5}C$

$q_1 = 1.10\times10^{-5}C$

11 Three charges were positioned as shown in the figure below. If $Q_1 = Q_2 = 8\mu C$ and $d = 0.5m$, determine q if the electric field at P is zero.



Solution



$\sqrt{2d^2 + d^2} = d\sqrt{5}$, $\tan\theta = \frac{2d}{d}$

$d = 0.5$

$E_1 = \frac{kq_1}{r^2} = \frac{9\times10^9 \times 8\times10^{-6}}{(d\sqrt{5})^2} = \frac{9\times10^9 \times 8\times10^{-6}}{(\sqrt{5}/2)^2} = 57600\,N/C$

$E_2 = \frac{kq_2}{r^2} = \frac{9\times10^9 \times (8\times10^{-6})}{(d\sqrt{5})^2} = 57600\,N/C$

$E_q = \frac{kq}{r^2} = \frac{9\times10^9 \times q}{(2d)^2} = \frac{9\times10^9}{1^2} = 9\times10^9 q\,N/C$

vector	θ	x component	y component
$E_1 = 57600\,N/C$	63.43	$57600\cos 63.43 = -25764$	$57600 \sin 63.43 = +51516.8$
$E_2 = 57600\,N/C$	63.43	$57600\cos 63.43 = +25764$	$57600 \sin 63.43 = +51516.8$
$E_q = 9\times10^9 q\,N/C$	90°	$9\times10^9 q \cos 90$ $\Sigma f_x = 0$	$9\times10^9 q \sin 90 = 9\times10^{10} q$ $\Sigma f_y = 103033.6 + 9\times10^9 q$

$E_{net} = \sqrt{(\Sigma f_x)^2 + (\Sigma f_y)^2}$ E_{net} at a point $= 0$

$\therefore\ 0 = \sqrt{0^2 + (103033.6 + 9\times10^9 q)^2}$

$0 = 103033.6 + 9\times10^9 q$

$q = \frac{-103033.6}{9\times10^9}$

$q = -1.14481\times10^{-5}$

$q = -11.4\times10^{-6}$

$q = -11.4\,NC$ //

x

3 State the formulation of the following identities of charge

i) volume charge density

$- \rho = \frac{dQ}{dv} \rightarrow dQ = \rho dv$

ii) surface charge density

$- \sigma = \frac{dQ}{dA} \rightarrow dQ = \sigma dA$

iii) Linear charge density

$- \lambda = \frac{dQ}{dL} \rightarrow dQ = \lambda dL$

i) Explain with appropriate equations, the electric potential difference.

Answer

Electric potential difference between two point in an electric field can be defined as the work done per unit charge against electrical force when a charge is transported from one point to the other. It is measured in volt (V) or Joule per coulomb (J/C)

Equation — $dW = F\,dl$ ... ①

$F = -q_0 E$ ... ②

Substituting equation ② in ① yields

$dW = -q_0 E\,dl$ ... ③

Then total work done in moving the test charge from A to B is:

$W(A \to B)_{AB} = -q_0 \int^{B} E\,dl$ ... ④

From the definition of electric potential difference, it follows that:

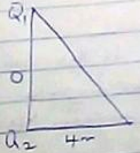
$V_B - V_A = \dfrac{W(A - B)_{AB}}{q_0}$ ... ⑤

Putting eq ④ in ⑤ yield

$V_B - V_A = -\int_A^B E\,dl$ ... ⑥

c) Two point charges $Q_1 = 10\mu C$ and $Q_2 = -2\mu C$ are arranged along the x-axis at $x = 0$ and $x - 4m$ respectively. Find the position along the x-axis where $v = 0$

solution



$V = 0 = \dfrac{1}{4\pi\varepsilon_0} = \left[\dfrac{Q_1}{r_1} + \dfrac{Q_2}{r_2}\right]$

$-2Q$ $10Q$

$4d$ ——————|—————— 0 ..... x-axis

∴ Between charges

$V_1 + V_2 = 0$ ∴ D = distance

$$\frac{k(-2Q)}{x-(4d)} + \frac{k\,10Q}{0-x}$$

$$\frac{-2}{x-4d} = \frac{10}{-x}$$

$2x = 10x - 40d$

$40d = 10x - 2x$

$40d = 8x$ divide through by 8

∴ $\cancel{\frac{8x}{8} = \frac{40}{8}}$ $5d = x$

$x = 5m$ ∴ the position where $v = 0$ = 5 metres.

4A what is magnetic flux?

Answer:

Magnetic flux through a surface is the surface integral of the normal Component of the magnetic field flux density 'B' passing through that surface. Magnetic flux is often denoted by ϕ or ϕ_B.

Finding cyclotron frequency.

$\Rightarrow$ $m = 9.11 \times 10^{-31} kg$

$R = 1.4 \times 10^{-7} m$

$T = 3.5 \times 10^{-1} T$

note: Angular speed (ω) - cyclotron-frequency

$\omega = \frac{qB}{m_p}$

Note $q = 1.6 \times 10^{-19} C$ substituting fig in formula

$\Rightarrow$

$$\omega = \frac{1.6 \times 10^{-19} C \times (0.35 T)}{9.11 \times 10^{-31} kg}$$

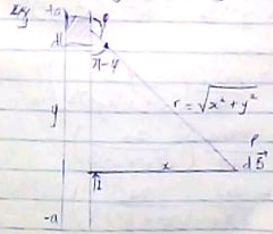
$\omega = 6.15 \times 10^{-52} rad/s$

C An electron with a mass of $9.11 \times 10^{-31} kg$ and charge of 1.6×10^{-19}, moves in a circular orbit of radius 1.4×10^{-7}, in a uniform magnetic field of 3.5×10^{-1} tesla perpendicular to the speed of light, will move at a cyclotron frequency of $6.15 \times 10^{-52} rad/s$

5 State Biot-Savart law.

Answer: The biot-savart law is based on the following observations for the magnetic field $d\vec{B}$ at at a point P associated with a length element $d\hat{l}$ of a wire carrying a steady current

(b)



A section of a straight current carrying conductor.

Applying the Biot-Savart law we find the magnitude of the field $d\vec{B}$

$$B = \frac{\mu_0 I}{4\pi}\int_{-a}^{a}\frac{dl\sin\varphi}{r^2}$$

$$\sin(\pi - \varphi) = \sin\theta$$

$$\therefore B = \frac{\mu_0 I}{4\pi}\int_{-a}^{a}\frac{dl\sin(\pi-\varphi)}{r^2}$$

From the diagram above, $r^2 = x^2 + y^2$ (Pythagoras Theorem)

$$B = \frac{\mu_0 I}{4\pi}\int_{-a}^{a}\frac{dl\sin(\pi-\varphi)}{x^2+y^2} \quad \text{--- eq①}$$

But $\sin(\pi-\varphi) = \frac{x}{\sqrt{x^2+y^2}} = \frac{x}{(x^2+y^2)^{1/2}}$ — eq②

Substituting eqn 2 into eq ① we have

$$B = \frac{\mu_0 I}{4\pi}\int_{-a}^{a} dl\,\frac{x}{(x^2+y^2)(x^2+y^2)^{1/2}}$$

$$B = \frac{\mu_0 I}{4\pi}\int_{-a}^{a} dl\,\frac{x}{(x^2+y^2)^{3/2}}$$

Recall $dl = dy$

$$B = \frac{\mu_0 I}{4\pi}\int_{-a}^{a}\frac{x}{(x^2+y^2)^{3/2}}dy$$

$$B = \frac{\mu_0 I x}{4\pi}\int_{-a}^{a}\frac{1}{(x^2+y^2)^{3/2}}dy \quad \text{--- eq③}$$

using special integrals

$$\int\frac{dy}{(x^2+y^2)^{3/2}} = \frac{1}{x^2}\cdot\frac{y}{(x^2+y^2)^{1/2}}$$

eqn ③ therefore becomes

$$B = \frac{\mu_0 I x}{4\pi}\left[\frac{y}{x^2(x^2+y^2)^{1/2}}\right]_{-a}^{a}$$

$$B = \frac{\mu_0 I x}{4\pi}\left(\frac{2a}{x^2(x^2+a^2)^{1/2}}\right)$$

$$B = \frac{\mu_0 I}{4\pi x}\left(\frac{2a}{(x^2+a^2)^{1/2}}\right)$$

when the length $2a$ of the conductor is very great in comparison to distance x from point P, we consider it infinitely long. That is, when a is much larger than x, $(x^2+a^2)^{1/2} \approx a$, as $a \longrightarrow \infty$ $\therefore B = \frac{\mu_0 I}{2\pi x}$