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19/MHS 10/003
OPTOMETRY

1 $y = \frac{1}{(x-2)}$

The function is defined for all real numbers except $x = 2$

Domain = Real numbers except $x = 2$

Co-domain = Real numbers except $y = 0$

2 If $K = \ln V$, differentiate K

$$\frac{dK}{dV} = \frac{d(\ln V)}{dV}$$

$$= \frac{1}{V}$$

$$\therefore \frac{dK}{dV} = \frac{1}{V}$$

3a $2x - 3y - 2 = 0$

$2x - 2 = 3y$

$$y = \frac{2x-2}{3}$$

5 $x^2 + y^2 = 4$

$y^2 = 4 - x^2$

$y = \sqrt{4 - x^2}$

4 $P = \sin^{-1} t$, find the derivative of P

$P = \sin^{-1} t$

$P = \frac{t}{\sin} \Rightarrow t = \sin p$ --- ①

$\frac{dt}{dp} = \cos p$

$\cos^2 p = 1 - \sin^2 p$

$\cos p = \sqrt{1 - \sin^2 p}$

$\sin p = t \therefore \sin^2 p = t^2$

$\therefore \cos p = \sqrt{1 - t^2}$

$$\frac{dp}{dt} = \frac{1}{\frac{dt}{dp}}$$

$$= \frac{1}{\quad}$$

$\therefore \frac{dp}{dt} = \frac{1}{\sqrt{1-t^2}}$

5 $f(x) = 2x^2 - 5$

$g(x) = 4x - 2$

$fog(x) = f(g(x))$

$= f(4x-2)$

$= 2(4x-2)^2 - 5$

$= 2(4x-2)(4x-2) - 5$

$= 2(16x^2 - 8x - 8x + 4) - 5$

$= 2(16x^2 - 16x + 4) - 5$

$= 32x^2 - 32x + 8 - 5$

$= 32x^2 - 32x + 3$

$gof(x) = g(f(x))$

$= g(2x^2 - 5)$

$= 4(2x^2 - 5) - 2$

$= 8x^2 - 20 - 2$

$= 8x - 22$

6 $f(x) = 3x^2 - 2x + 1 = 0$; show

$f_e(x) + f_o(x) = f(x)$

$f_e(x) = \frac{f(x) + (\cancel{-f}(x))}{2}$

$f(-x) = \cancel{-(3x^2 - 2x + 1)}$

$= -3x^2 + 2x + 1$

$f_e(x) = \frac{3x^2 - 2x + 1 + (-3x^2 + 2x + 1)}{2}$

$= \frac{(3x^2 - 3x^2 - 2x + 2x + 1 + 1)}{2}$

$= \frac{2}{2}$

$f_e(x) = 1$

$f_o(x) = \frac{f(x) - f(-x)}{2}$

$$= \frac{3x^2 - 2x + 1 - (-3x^2 + 2x + 1)}{2}$$

$$= \frac{3x^2 - 2x + 1 + 3x^2 - 2x - 1}{2}$$

$$= \frac{6x^2 - 4x}{2}$$

$$= \frac{2(3x^2 - 2x)}{2}$$

$f_e(x) = 3x^2 - 2x$

$f_e(x) + f_o(x) = f(x)$

$3x^2 - 2x + 1 = f(x)$

$\therefore f(x) = 3x - 2x + 1$

2 $y = \cos x$; differentiate from first principle

$y + \delta y = \cos(x + \delta x)$

$\delta y = \cos(x + \delta x) - y$; but $y = \cos x$

$\therefore \delta y = \cos(x + \delta x) - \cos x$ - - - (1)

$\cos(A + B) - \cos(A - B) = -2\sin A \sin B$ - - - (2)

compare 1 and 2

$A + B = x + \delta x$

$A - B = x$

$\cancel{A = x + \delta x}$

$A = x + \frac{\delta x}{2}$

$B = \frac{\delta x}{2}$

compare 1 and 2:

$\cos(x + \delta x) - \cos x = -2\sin\left(x + \frac{\delta x}{2}\right)\sin\left(\frac{\delta x}{2}\right)$

$\delta y = -2\sin\left(x + \frac{\delta x}{2}\right)\sin\left(\frac{\delta x}{2}\right)$

$$\frac{\delta y}{\delta x} = \frac{-2\sin\left(x + \frac{\delta x}{2}\right)\sin\left(\frac{\delta x}{2}\right)}{\delta x}$$

$$= -\sin\left(x + \frac{\delta x}{2}\right)\frac{\sin\left(\frac{\delta x}{2}\right)}{\frac{\delta x}{2}}$$

$$\lim_{\delta x \to 0} \frac{\sin\left(\frac{\delta x}{2}\right)}{\left(\frac{\delta x}{2}\right)} = 1$$

$$\lim \frac{\delta y}{\delta x} = \lim_{\delta x \to 0} -\sin\left(x + \frac{\delta x}{2}\right) \frac{\sin\left(\frac{\delta x}{2}\right)}{\frac{\delta x}{2}}$$

$$= -\sin(x + 0)$$

$$= -\sin x$$

yorn, $\lim_{\delta x \to 0} \frac{\delta y}{\delta x} = \frac{dy}{dx} = -\sin x$

8 If $y = 3t^2$, $x = \frac{1}{t^2}$, find $\frac{dy}{dx}$

$y = 3t^2$ $\qquad$ $x = \frac{1}{t^2} = t^{-2}$

$\frac{dy}{dt} = 6t$

$\frac{dx}{dt} = -2t^{-3}$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$$

$$= \frac{6t}{-2t^{-3}}$$

$$= 6t \times \frac{1}{-2t^{-3}}$$

$$= 6t \times \frac{t^3}{-2}$$

$$= \frac{6t^4}{-2}$$

$$= -3t^4$$

9 ~~d~~ If $y = x^2 \cos 2x\, e^{4x}$, find $\frac{dy}{dx}$

$$\ln y = \ln(x^2 \cos 2x\, e^{4x})$$

$$\ln y = \ln(x^2) + \ln \cos 2x + \ln e^{4x}$$

$$\frac{d(\ln y)}{dx} = \frac{d(\ln x^2)}{dx} + \frac{d(\cos 2x)}{dx} + \frac{d}{dx}(\ln e^{4x})$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{1}{x^2}(2x) + \frac{1}{\cos 2x}(-2\sin 2x) + \frac{1}{e^{4x}} 4e^{4x}$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{2}{x} - 2\tan 2x + 4$$

$$\frac{dy}{dx} = y\left(\frac{2}{x} - 2\tan 2x + 4\right)$$

$$= x^2 \cos 2x\, e^{4x}\left(\frac{2}{x} - 2\tan 2x + 4\right)$$

10 $y = \sin(3x^3 + 5)$; find the derivative of y.

$$y = \sin(3x^3 + 5) \qquad \cancel{\frac{dy}{du}} \qquad y = \sin u$$

$$u = 3x^3 + 5 \qquad \frac{dy}{du} = \cos u$$

$$\frac{du}{dx} = 9x^2$$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$$= \cos u \times 9x^2$$

$$= 9x^2 \cos u$$

$$= 9x^2 \cos(3x^3 + 5)$$