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15/ENG07/019
Petroleum Engineering
PTE516: Multi-Phase Flow In Pipeline
Assignment.

1. Horizontal Multiphase Flow Regimes:

Multiphase flow in horizontal pipes differs from that in vertical pipes. Due to the Potential Energy Contribution in horizontal flow, the Flow regime has no significant effect on the pressure drop in horizontal Flow. However, Certain Correlations used in Multiphase horizontal Flow ~~regimes~~ Consider the Flow regimes.

Horizontal Flow regimes Can be Classified into;
- Segregated Flow : Here, the two phases are separated for most of the time.
- Intermittent Flow : Where the gas and Liquid phases are alternating.
- Distributive Flow: The flow regime where one phase is dispersed in the other.

The Segregated Flow is further divided into;
(a) Stratified Smooth - This occurs at low flow rate.
(b) Stratified wavy - This occurs at high flow rate.
(c) Annular - This occurs at high gas and high Liquid flow rate.

The Intermittent Flow regime is divided into;
(a) Slug (High liquid slugs and high velocity gas bubbles) &
(b) Plug Flow

The Distributive Flow is divided into; (a) Bubble, (b) Mist (c) Dispersed bubble.

Flow regimes are Predicted with maps like;
- Baker (1953) which was modified to Scott (1963).
- Mandhane et al (1974)
- Beggs and Bills Correlation; which Plots Fronde Numer against Liquid Fraction.
- Taitel and Duckler 1976; A theoretical model used to generate Flow regime maps for particular fluids and Pipe size.

(2) Question 10-4

Data Given:

$q_L = 500 bbl/d$ $P = 1000 Psig$ $D = 2$ inches $P_{pc} = 667 Psi$

$GOR = 1000 scf/bbl$ $Z = f\left(\frac{580}{395}, \frac{1000}{667}\right) = 0.85$ $\rho_g = ?$

$\sigma_L = 20 dynes/cm$ $\gamma_o = 32°API$ $\mu_L = 2.0cp$ $\rho_L = ?$

$T = 120°F = 580°R$ $\gamma_g = 0.71$ $T_{pc} = 395°R$ $\mu_g = 0.0131$

Using Baker's Correlation.

$$A = \frac{\pi}{4} \times \left(\frac{2}{12}\right)^2 \quad A = 0.02182 ft^2$$

$$\rho_g = \frac{28.97 \gamma_g P}{ZRT}$$

$$\rho_g = \frac{28.97 \times 0.71 \times 1000}{0.85 \times 10.73 \times 580} = 3.89 lbm/ft^3$$

$$\gamma_o = \frac{141.5}{32 + 131.5} = 0.865$$

$$\rho_o = 0.865 \times 62.4 = 54 lbm/ft^3$$

$$\therefore U_{SL} = \frac{q_L}{A} = \frac{500 \times 5.615}{86400 \times 0.02182} = 1.4392 ft/s.$$

$$U_{sg} = \frac{4}{\pi \times \left(\frac{2}{12}\right)^2} \times \frac{500000}{86400} \times 0.85 \times \frac{580}{520} \times \frac{14.7}{1000} = 3.697 ft/s$$

$$\lambda = \left[\left(\frac{\rho_g}{0.075}\right)\left(\frac{\rho_L}{62.4}\right)\right]^{1/2}$$

$$\lambda = \left[\left(\frac{3.89}{0.075}\right)\left(\frac{54}{62.4}\right)\right]^{1/2}$$

$$\lambda = 6.6996$$

$$\phi = \frac{73}{\sigma_L}\left[\mu_L\left(\frac{62.4}{\rho_L}\right)^2\right]^{1/3} = \frac{73}{20}\left[2\left(\frac{62.4}{54}\right)^2\right]^{1/3}$$

$$\phi = 5.064$$

$$G_g = U_{sg} \times \rho_g = 3.697 \times 3.89 \times 3600 = 5.1773 \times 10^4.$$

$$G_L = U_{SL} \times \rho_L = 1.4892 \times 54 \times 3600 = 2.895 \times 10^5$$

$$\frac{G_g}{\lambda} = \frac{5.1773 \times 10^4}{6.6996} = \underline{\underline{7.728 \times 10^3}}$$

$$\frac{G_L \lambda \phi}{G_g} = \frac{2.895 \times 10^5 \times 6.6996 \times 5.064}{5.1773 \times 10^4} = \underline{\underline{189.71}}$$

Question 2 Continues....

Slug Flow, That is flow is a fn of $\frac{G_g}{\lambda}$, $\frac{G_L \lambda \phi}{G_g}$ From Baker's map.

Mandhane Flow (U_{sL}, U_{sg})

$U_{sL} = 1.4892 \text{ ft/s}$

$U_{sg} = 3.697 \text{ ft/s}$

From Mandhane Flow map.

Flow regime = Slug Flow

~~Begg's~~

Begg's and Bills

Flow = $f(N_{FR}, \lambda_L)$ Function of Froude Number and liquid fraction.

$N_{FR} = \frac{U_m^2}{gD}$

$U_m = 1.4592 + 3.697 = 5.1862 \text{ ft/s}$

$g = 32.19 \text{ ft/sec}^2$

$D = \left(\frac{2}{12}\right) \text{ ft}$

$\Rightarrow N_{FR} = \frac{5.1862^2}{32.17 \times \left(\frac{2}{12}\right)}$

$N_{FR} = 5.0165$

$\Rightarrow \lambda_L = \frac{U_{sL}}{U_{sL} + U_{sg}} = \frac{1.4892}{1.4892 + 3.697}$

$\lambda_L = 0.287$

Flow Regime = Intermittent.

Question 10-6

Data Given

$q_o = 4000 \, bbl/D$ $\quad \varepsilon = 0.001$ $\quad \mu_L = 2.0 \, cp$

$GOR = 500 \, scf/bbl$ $\quad P = 200 \, Psia$ $\quad \mu_g = 0.0131 \, cp$

$D = 3''$ $\quad \sigma_L = 20 \, dyne/cm$ $\quad T_{pc} = 395°R$

$T = 150°F = 610°R$ $\quad \gamma_o = 32° API$ $\quad P_{pc} = 667 \, Psi$

Derived Parameters

$Z = f\left(\frac{610}{395}, \frac{200}{667}\right) = 0.97$; $\gamma_o = \frac{141.5}{32 + 131.5} = 0.865$

$Area = \left(\frac{\pi}{4}\right)\left(\frac{3}{12}\right)^{2} = 0.0491 ft^2$; $\rho_o = 0.865 \times 62.4 = 54 \, lbm/ft^3$

$\rho_g = \frac{28.97 \gamma_g P}{ZRT} = \frac{28.97 \times 0.71 \times 200}{0.97 \times 10.73 \times 610} = 0.648 \, lbm/ft^3$

$q_g = GOR \times q_L = 500 \times 4000 = 2 \times 10^6 ft^3/d$

Beggs and Bill Method

Flow regime Calculation.

$U_{SL} = \frac{q_L}{A} = \frac{4000 \times 5.615}{86400 \times 0.0491} = 5.2944 \, ft/s$

$U_{sg} = \frac{4}{\pi D^2} \times q \times Z \times \left(\frac{T}{T_{sc}}\right) \times \left(\frac{P_{sc}}{P}\right)$

$= \frac{4}{\pi \times \left(\frac{3}{12}\right)^2} \times \frac{2 \times 10^6}{86400} \times 0.97 \times \frac{610}{520} \times \frac{14.7}{200}$

$U_{sg} = 39.4395 \, ft/s$

$U_m = U_{SL} + U_{sg} = 39.4395 + 5.2944 = 44.7339$

$\lambda_L = \frac{5.2944}{44.7339} = 0.11835$

$N_{FR} = \frac{U_m^2}{gD} = \frac{44.7339^2}{32.17 \times \left(\frac{3}{12}\right)} = 248.918$

$L_1 = 316 \times (0.11835)^{0.502} = 165.876$

$L_2 = 0.000925 \, (0.11835)^{-2.4684} = 0.1795$

$L_3 = 0.1 \times (0.11835)^{-1.4516} = 2.2151$

$L_4 = 0.5 \, (0.11835)^{-5.738} = 104{,}023.273$

Flow is distributive since $\lambda_L < 0.4$ and $N_{FR} \geq L_1$

Question 3 continues...

Hold-Up Calculation

$y_L = y_{L0}\psi$

$y_{L0} = \frac{a\lambda_L^b}{N_{FR}^c}$

$y_{L0} = \frac{1.065\times(0.11835)^{0.5824}}{248.818^{0.0609}}$

$y_{L0} = \frac{0.3073}{1.3993} = 0.21961$

$\rho_m = 54\times0.11835 + 0.648\times0.88165 = 6.9622\ lbm/ft^3$

$\mu_m = \mu_L\lambda_L + \mu_g\lambda_g = 2\times0.11835 + 0.0131\times0.88165 = 0.24825$

$N_{Rem} = \frac{\rho_m V_m D}{\mu_m} = \frac{6.9622\times44.7339\times(3/12)}{6.72\times10^{-4}\times0.24825}$

$N_{Rem} \simeq 4.7\times10^5$

$f_n = 0.006$

Calculating for x, S, f_{tp}

$x = \frac{\lambda_L}{y_{L0}^2} = \frac{0.11835}{0.21961^2} = 2.434$

$S = \frac{\ln(x)}{[-0.0528 + 3.182\ln(x) - 0.8725[\ln(x)] + 0.01853[\ln(x)]^4]}$

$S = 0.4248$

$f_{tp} = f_n \cdot e^S = 0.006\times e^{0.4248}$

$f_{tp} = 0.009176$

Frictional Pressure gradient Calculation.

$\left(\frac{dP}{dz}\right)_F = \frac{2f_{tp}\rho_m V_m^2}{g_c D} = \frac{2\times0.009176\times6.9622\times44.7339^2}{32.17\times3/12}$

$\frac{dP}{dz} = 31.792\ lbf/ft^3$

Converting to ~~ft~~ Psi/ft

$\frac{dP}{dz} = 31.792\frac{lbf}{ft^3}\times\frac{1ft^2}{144\ in^2}$

$\frac{dP}{dz} = 0.221\ Psi/ft$

Question 3 Contin...

Eaton Correlation

$\dot{m}_L = q_L \rho_L$ Mass flow rate.

$q_L = 4000 \frac{bbl}{d} \times \frac{5.615 ft^3}{86400 \text{ sec}} = 0.26 ft^3/s$

$\dot{m}_L = 0.26 ft^3/s \times 54 lbm/ft^3 = 14.038 lbm/s$

$\dot{m}_g = q_g \rho_g$

$\cancel{q_g = 2\times10^6 \frac{ft^3}{d} \times \frac{1 d}{86400 \text{ sec}}}$

$q_g = A \times u_{sg} = 0.0491 \times 39.4395 = 1.936$

$\dot{m}_g = 1.936 \times 0.648 = 1.255 lbm/s.$

$\dot{m}_m = \dot{m}_L + \dot{m}_g = 14.038 + 1.255$

$\dot{m}_m = 15.293 lbm/s$

Gas Velocity (μ_g)

$\mu_g = 0.0131 \times 6.72 \times 10^{-4}$

$= 8.8 \times 10^{-6} lbm/ft\text{-}sec.$

Calculating f;

$\cancel{\frac{0.057 (\dot{m}_g \dot{m}_m)^{0.5}}{\mu_g D^{2.25}}}$

From fig 10.6; $f \left(\frac{\dot{m}_L}{\dot{m}_m}\right)^{0.1} = 0.02$

$$f = \frac{0.02}{\left(\frac{14.038}{15.293}\right)^{0.1}} = 0.0202$$

$$\left(\frac{dP}{dx}\right)_f = \frac{f \rho_m u_m^2}{2 g_c D} = \frac{0.0202 \times 6.9622 \times 44.7339^2}{2 \times 32.17 \times (3/12)}$$

$$\frac{dP}{dx} = 0.122 \text{ Psi/ft}$$

Duckler Correlation

$$\frac{dP}{dx} = \left(\frac{dP}{dx}\right)_F + \left(\frac{dP}{dx}\right)_{K.E}$$

Frictional Pressure drop

$$\left(\frac{dP}{dx}\right)_F = \frac{f \rho_K u_m^2}{2 g_c D}$$

Continues...

$$\rho_K = \frac{\rho_L \lambda_L^2}{y_L} + \frac{\rho_g \lambda_g^2}{y_g} \quad \text{and} \quad N_{Re_K} = \frac{\rho_K u_m D}{\mu_m} = N_{Re_m}\left(\frac{\rho_K}{\rho_m}\right)$$

Assuming $\lambda_L = y_L$; $\rho_K = \rho_m$, $N_{Re_m} = N_{Re_K}$

$$\lambda_L = y_L = 0.11835$$

$$\rho_K = \frac{54 \times 0.11835^2}{0.11835} + \frac{0.648 \times 0.88165}{0.88165}$$

$$\rho_K = 6.962\ lbm/ft^3$$

$$N_{Re_K} = 4.7 \times 10^5 \left(\frac{6.962}{6.9622}\right) = 4.7 \times 10^5$$

$$f_n = 0.0056 \times 0.5 \left(N_{Re_K}\right)^{-0.32}$$

$$f_n = 0.013$$

$$\frac{f}{f_n} = 1 - \left[\frac{\ln \lambda_L}{1.281 + 0.478 \ln\lambda_L + 0.444 (\ln\lambda_L)^2 + 0.094 (\ln\lambda_L)^3 + 0.00843 (\ln\lambda_L)^4}\right]$$

$$\frac{f}{f_n} = 1 - (-1.3818)$$

$$f = f_n \times 2.3818 = 0.013 \times 2.3818$$

$$f = 0.031$$

$$\left(\frac{\partial P}{\partial x}\right)_F = \frac{f \rho_K N_m^2}{2 g_c D}$$

$$= \frac{0.013 \times 6.962 \times 44.7339^2}{2 \times 32.17 \times 0.25}$$

$$= 11.26\ lbf/ft^3 \Rightarrow 0.078\ Psi/ft$$

1 Continues - - - -

4 Flow through Restriction:

Restricted flow refers to flow of fluid under a choke used to control the flow rate due to many factors like; production control, prevention of coning and sand production.

Fluid under choke may be accelerated to reach sonic velocity in the throat of the choke. This is the critical condition and changes in downstream pressure of choke do not affect flow rate.

- For Single-Phase liquid Flow through Restriction;

It is very rare, because the flowing pressure is below bubble point pressure. However, in the case of it; Flow rate is given in terms of pressure drop across the choke as;

$$q = CA\sqrt{\frac{2g_c \Delta P}{\rho}}$$

Where C = Flow coefficient of choke & A = Cross sectional Area

$$q = 22800 (D_2)^2 \sqrt{\frac{\Delta P}{\rho}}$$

D = choke diameter in inches.

- Single-Phase gas Flow through Restriction.

Gas is compressible fluid, thus its expansion is an important factor to be considered.

For isentropic flow of ideal gas under restriction, the flow rate is given as;

$$q_g = \frac{\pi}{4} D_2^2 \pi \frac{T_{sc}}{P_{sc}} \alpha \sqrt{\left(\frac{2g_c R}{28.97 \gamma_g T_1}\right)\left(\frac{\gamma}{\gamma - 1}\right)\left[\left(\frac{P_2}{P_1}\right)^{2/\gamma} - \left(\frac{P_2}{P_1}\right)^{(\gamma+1)/\gamma}\right]}$$

In oil field units;

$$q_g = 3.505 D_{64}^2 \left(\frac{P_1}{P_{sc}}\right) \alpha \sqrt{\left(\frac{1}{\gamma_g T_1}\right)\left(\frac{\gamma}{\gamma - 1}\right)\left[\left(\frac{P_2}{P_1}\right)^{2/\gamma} - \left(\frac{P_2}{P_1}\right)^{(\gamma+1)/\gamma}\right]}$$

Both equation apply when pressure ratio $\geq$ the critical pressure ratio given as

$$\left(\frac{P_2}{P_1}\right)_c = \left(\frac{2}{\gamma + 1}\right)^{\gamma/(\gamma - 1)}$$