

## Question One.

Given data.

$$q_o = 500 \text{ bbl/day}$$

$$D = 2 \text{ inches}$$

$$\text{Temp} = 120^\circ\text{F} + 460 = 580^\circ\text{R}$$

$$\text{Pressure} = 1000 \text{ psia}$$

$$q_L = ?$$

$$\text{GOR} = 1000 \text{ scf/bbl}$$

$$\text{Area} = ?$$

Step 1  $\rightarrow$  Calculate Area.

$$\text{Using } A = \frac{\pi D^2}{4} = \frac{3.143 \times \left(\frac{2}{12}\right)^2}{4} = 0.02182 \text{ ft}^2$$

Step 2  $\rightarrow$  Calculate  $U_{sc}$ ,  $U_{sg}$ ,  $U_m$ .

$$\text{where } U_{sc} = \frac{q_L}{A}$$

$$U_{sc} = \frac{500 \times 5.614 \text{ ft}^3}{86400 \text{ sec}} = \frac{1.48 \text{ ft}^3/\text{s}}{0.02182}$$

$$U_{sg} = \frac{q_g}{A} = \frac{4}{\pi D^2} \left[ q \times Z \right] \left[ \frac{T}{T_{sc}} \right] \times \left[ \frac{P_{sc}}{P} \right]$$

To calculate flowrate of gas when GOR and flowrate of oil is given.

$$\text{GOR} = 1000 \text{ scf/bbl} = \frac{\text{ft}^3}{\text{bbl}} = 1000$$

$$\frac{\text{ft}^3}{\text{day}} = 1000$$

To get  $G_L$  and  $G_g$

$$G_L = 1166 P_L \quad \dots \textcircled{*}$$

$$G_g = 1166 P_g \quad \dots \textcircled{*}$$

$$G_L = \left( \frac{1.487t}{s} \right) \left( \frac{53.96 \text{ km}}{ft^3} \right) \left( \frac{3600 \text{ sec}}{hr} \right)$$

$$G_L = \underline{\underline{287498.88 \text{ km/hr-ft}^2}}$$

$$G_g = \left( \frac{3.697t}{s} \right) \left( \frac{3.88 \text{ km}}{ft^3} \right) \left( \frac{3600 \text{ sec}}{hr} \right)$$

$$G_g = \underline{\underline{51,541.92 \text{ km/hr-ft}^2}}$$

To get  $\lambda$  and  $\phi$

$$\lambda = \left[ \left( \frac{P_g}{0.075} \right) \left( \frac{P_L}{62.4} \right) \right]^{\frac{1}{2}} \quad \dots \textcircled{*}$$

$$\phi = \frac{73}{20} \left[ \frac{1166 \left( \frac{62.4}{P_L} \right)^2}{53.96} \right]^{\frac{1}{3}} \quad \dots \textcircled{*}$$

$$\lambda = \left[ \left( \frac{3.88}{0.075} \right) \left( \frac{53.96}{62.4} \right) \right]^{\frac{1}{2}}$$

$$\lambda = \sqrt{(44.7360)} = \underline{\underline{6.688}}$$

$$\phi = \frac{73}{20} \left[ \frac{1166 \left( \frac{62.4}{53.96} \right)^2}{53.96} \right]^{\frac{1}{3}}$$

$$\phi = \frac{73}{20} (2.67457)^{\frac{1}{3}} = \underline{\underline{5.1}}$$

$$\frac{Ft^3}{day} \times \frac{day}{667} = 1000$$

$$\frac{Ft^3}{day} \times \frac{1}{\frac{667}{day}} = 1000$$

$$\frac{Ft^3}{day} \times \frac{1}{500} = 1000$$

$$\frac{Ft^3}{day} \times 0.002 = 1000$$

$$82 Ft^2 \frac{Ft^3}{day} = \frac{1000}{0.002} = \underline{\underline{500,000}} \text{ (Flowrate)}$$

\* Since  $Z$  was not given.  
Use the  $Z$ -Factor Chart.

$$\frac{T}{T_{pc}} = \frac{580}{395} = 1.47, \quad \frac{P}{P_{pc}} = \frac{1000}{667}$$

From the Chart.

$$Z = 0.85$$

Therefore.

$$U_{sg} = \frac{4}{\pi \left(\frac{7}{2}\right)^2 Ft^2} \times \left( \frac{500,000 \times 0.850}{86400} \right) \left( \frac{580}{520} \right) \left( \frac{14.7}{1000} \right)$$

$$U_{sg} = \frac{4}{0.08726} \times \left( \frac{425000}{86400} \right) \left( \frac{580}{520} \right) \left( \frac{14.7}{1000} \right)$$

$$U_{sg} = \frac{14,494,200,000}{3,920,417,280} = \underline{\underline{4.06}}$$

$$N_{Rem} = \frac{\rho_m U_m D}{\mu_m} = \frac{7.05084 \times 44.33 \times \frac{3}{12}}{0.250 \times 6.72 \times 10^{-7}}$$

$$N_{Rem} = 461,433.14377$$

then calculate for  $\alpha$ ,  $S$ ,  $F_{TP}$

$$\alpha = \frac{\lambda_L}{\gamma L_0^2} = \frac{0.12}{(0.22)^2} = \underline{\underline{2.5}}$$

$$S = \ln(\alpha)$$

$$[-0.0523 + 3.182 \ln(\alpha) - 0.8725 [\ln(\alpha)]^2 + 0.01853 \ln(\alpha)]$$

$$S = 0.92$$

$$[-0.0523 + 3.182(0.92) - 0.8725(0.92)^2 + 0.01853(0.92)]$$

$$S = 0.4279$$

$$F_{TP} = \tau_m e^S = 0.006 \times e^{0.4279} = \underline{\underline{0.009176}}$$

step 5  $\rightarrow$  calculate  $(\frac{dp}{dz})_T$

$$\frac{dp}{dz} = \frac{2 F_{TP} \rho_m U_m^2}{g_c D}$$

$$\frac{dp}{dz} = \frac{2 \times 0.009176 \times 7.05084 \times (44.33)^2}{32.17 \times (\frac{3}{12})}$$

$$\frac{dp}{dz} = 32.0116 \text{ ft}^2 = 32.0116 \times \frac{1 \text{ ft}}{144 \text{ in}^2}$$

$$\frac{dp}{dz} = 0.2229 \text{ psi/ft}$$

From Fig 10.6

$$F \left( \frac{mL}{\mu m} \right)^{0.1} = 0.02$$

$$F = 0.02 \left( \frac{14.03}{15.283} \right)^{0.1} = \underline{\underline{0.0201}}$$

$$\left( \frac{dP}{dx} \right) = \frac{F \rho_m U_m^2}{2g_c D} \quad \text{[Pressure gradient]}$$

$$= \frac{0.0201 \times 7.050 \times (44.33)^2}{2 \times 32.17 \times 0.25}$$

$$= \frac{17.0167 \times 10^3}{144} \times \frac{176^2}{14416}$$

$$= \underline{\underline{0.12 \text{ psi/ft}}}$$

For Dukler Correlation.

STEP 1  $\rightarrow$  Assume  $\gamma_L = \rho_L$ ,  $\rho_g = \rho_m$

$$N_{ReL} = N_{ReM}$$

$$\gamma_L = \lambda_L = 0.15$$

$$\rho_{L1} = \rho_m =$$

$$\rho_{L1} = \frac{\rho_L \lambda_L^2}{\gamma_L} + \frac{\rho_g \lambda_g^2}{\gamma_g}$$

$$\gamma_g = 1 - \gamma_L$$

Question 8

$$\frac{1}{f_{10}} = 1 - \frac{\ln(0.12)}{0.2665 + 1.9780} = 0.8959 + 0.16971$$

$$\frac{F}{f_{10}} = - \frac{2.12}{1.51836}$$

$$\frac{F}{f_{10}} = 1 - \left( \frac{2.12}{1.51836} \right)$$

$$F = \frac{-2.12 \times 0.00630}{1.51836}$$

$$\frac{F}{f_{10}} = \frac{1}{1 + \frac{2.12}{1.51836}}$$

$$\frac{F}{f_{10}} = \frac{1.51836}{1.51836 + 2.12}$$

$$1.51836 F = (1.51836 + 2.12) 0.00630$$

$$F = \underline{\underline{0.023}}$$

Pressure gradient

$$\left( \frac{dP}{dx} \right)_F = \frac{F \rho K U_m^2}{2g_c D} = \frac{0.023 \times 5.77 \times 44.33^2}{2 \times 32.17 \times 0.25}$$

$$\left( \frac{dP}{dx} \right)_F = \underline{\underline{16.2185}} \quad \underline{\underline{0.113 \text{ Psi/ft}}}$$

$$Y_L = 0.15$$

$$\lambda_g = \frac{U_{gs}}{G_{max}} =$$

$$P_{15} = \frac{55.93 \times 0.12^2}{0.15} + \frac{0.654 \times 0.2805}{(1 - 0.15)}$$

$$P_{15} = 5.1773 + 0.5965 = 5.7738 \text{ km/h}$$

Next calculate  $NR_{ek}$ .

$$NR_{ek} = \frac{P_k U_m D}{M_m} = \frac{19.724 \times 44.33 \times 0.25}{0.0250}$$

$$NR_{ek} = \frac{NR_{em} U_m}{M_m} =$$

$$460433.1437 \times \frac{44.33}{0.0250}$$

$$NR_{ek} = 818213250.4088$$

Next calculate  $F_n$ .

$$F_n = 0.0056 + 0.5(NR_{ek})^{-0.32}$$

$$= 0.0056 + 0.5(818213250.4088)^{-0.32}$$

$$F_n = 0.00680$$

$$\frac{F}{F_n} = \frac{\ln(\lambda_L)}{1.281 + 0.478 \ln(\lambda_L) + 0.44[\ln(\lambda_L)]^2 + 0.044 \ln(\lambda_L)^3 + 0.00843[\ln(\lambda_L)]^4}$$

To get Beggs Brill Reading, Use the Formulae

$$NFR = \frac{U_m^2}{g \cdot D}, \quad \lambda_1 = \frac{U_{sc}}{U_m}$$

$$NFR = \frac{(44.33)^2}{32.17 \times (\frac{3}{12})} = \frac{1965.1489}{8.0425} = 244.3$$

$$\lambda_2 = \frac{U_{sc}}{U_m} = \frac{5.30}{44.33} = 0.12$$

Therefore the Reading is

~~to get~~

STEP 3 → Calculate the holdup for horizontal flow. Using the Formulae

Recall that:

$$L_1 = 316 \lambda_1^{0.302}$$

$$L_2 = 0.0009252 \lambda_1^{-2.4684}$$

$$L_3 = 0.10 \lambda_1^{-1.4516}$$

$$L_4 = 0.5 \lambda_1^{-6.738}$$

$$L_1 = 316 (0.12)^{0.302} = 166.57$$

$$L_2 = 0.0009252 \times (0.12)^{-2.4684} = 0.17345$$

$$L_3 = 0.10 \times (0.12)^{-1.4516} = 2.171$$

$$L_4 = 0.5 \times (0.12)^{-6.738} = 800,657.48$$

The flow regime transition is

circulated flow because

So therefore, from the holdup constants table

$$a = 1.065, \quad b = 0.5824, \quad c = 0.0609.$$

$$Y_{10} = \frac{a L_1^b}{K_1 T^c} = \frac{1.065 \times (0.12)^{0.5824}}{(244.34)^{0.0609}}$$

$$Y_{10} = \frac{0.30978812}{1.39775} = \underline{\underline{0.22163}}$$

Step 4 → Calculate the no-slip friction factor based on the mixture Reynolds number

$$\rho_m = \rho_L \lambda_L + \rho_g \lambda_g$$

$$\lambda_L = 0.12, \quad \lambda_g = \frac{U_{sg}}{U_m} = \frac{39.033}{44.33} = \underline{\underline{0.8805}}$$

$$\rho_L = 53.96 \text{ lbm/ft}^3$$

$$\rho_g = \frac{2.7 \times \rho_g \times P}{Z \times T} = \frac{2.7 \times 0.71 \times 200}{0.96 \times 610}$$

$$\rho_g = 0.654 \text{ lbm/ft}^3$$

$$\rho_m = 53.96 \times 0.12 + 0.654 \times 0.8805$$

$$\rho_m = 6.475 + 0.5758 = \underline{\underline{7.050847 \text{ (lbm/ft}^3)}}}$$

\* Calculate Reynolds number using.

$$Re = \frac{\rho_m U_m}{\mu_m}$$

$$U_m = \mu_L \lambda_L + \mu_g \lambda_g$$

$$U_m = 2 \times 0.12 + 0.0131 \times 0.8805$$

$$U_m = 0.24 + 0.011534 = \underline{\underline{0.252}}$$

$$GOR = 500 \text{ sec} / 661$$

$$\frac{Ft^3}{\text{day}} = 500$$

$$\frac{Ft^3}{\text{day}} \times \frac{1}{\frac{661}{\text{day}}} = 500$$

$$\frac{Ft^3}{\text{day}} \times \frac{1}{4000} = 500$$

$$\frac{Ft^3}{\text{day}} \times 0.00025 = 500$$

$$\frac{Ft^3}{\text{day}} = \frac{500}{0.00025} = 2000,000$$

To get Z-Factor

$$\frac{T}{4pc} = \frac{610}{395} = 1.54 \quad \frac{P}{P_R} = \frac{200}{667} = 0.30$$

From the chart  $Z = 0.96$ .

$$U_{sg} = \frac{4}{\pi d^2} \left[ 9Z \left[ \frac{T}{4pc} \right] \frac{P_{ss}/P}{\left[ \frac{610}{520} \right] \left[ \frac{14.7}{200} \right]} \right]$$

$$U_{sg} = \frac{4}{\pi \left( \frac{3}{16} \right)^2} \times \left[ \frac{2 \times 10^6 \times 0.96}{86400} \right] \left[ \frac{610}{520} \right] \left[ \frac{14.7}{200} \right]$$

$$U_{sg} = 68,866,560,000 = 39.038 \text{ Ft/sec}$$

1,764,318,434.25603

$$U_{m} = U_{sg} + U_{sl} = 39.03 + 5.80 = 44.83$$

The coordinates for the Baker map are

$$\frac{Gg}{\lambda} = \frac{51541.92}{6.688} = \underline{\underline{7706.63}}$$

$$\frac{G\lambda\phi}{Gg} = \frac{287498.88 \times 6.688 \times 51}{51541.92} = \underline{\underline{190.26}}$$

\* Reading from the Baker map, the flow regime is predicted to be slug flow

\* Reading from the Mandhane map using  $U_{sl} = 1.487 \text{ ft/s}$ ,  $U_{sg} = 3.677 \text{ ft/sec}$  is predicted to be slug flow.

\* Reading from the Beggs and Brill map using the formulae

$$NF_2 = \frac{U_m^2}{gD}, \quad \lambda_c = \frac{U_{sl}}{U_m}$$

$$NF_2 = \frac{(5.176)^2}{32.17 \times \left(\frac{2}{12}\right)}$$

where  $g$  is constant  $32.17$ .

$$NF_2 = \frac{26.79076}{5.3616} = \underline{\underline{4.99}}$$

$$\lambda_L = \frac{U_{sl}}{U_m} = \frac{1.48}{5.176} = \underline{\underline{0.286}}$$

Seggs and Brills map is predicted to be intermittent

Question TWO.

Given data

$$q_o = 4000 \text{ bbl/day}$$

$$q_g = ?$$

$$D = 3 \text{ inches}$$

$$\sigma = 20 \text{ dynes/cm}$$

$$\text{Temp} = 150^\circ\text{F} + 460 = 610^\circ\text{R}$$

$$\text{Pressure} = 200 \text{ psia}$$

$$\text{GOR} = 500 \text{ scf/bbl}$$

Step 1  $\rightarrow$  Calculate the Area.

$$\text{Using } A = \pi d^2/4 = \frac{\pi \times (3/12)^2}{4} = \underline{\underline{0.049 \text{ ft}^2}}$$

Step 2  $\rightarrow$  Calculate  $U_{sl}$ ,  $U_{sg}$ ,  $U_m$

$$U_{sl} = \frac{q_o}{A} = \frac{4000 \times 5.614 \text{ ft}^3}{86400 \text{ sec}} = \underline{\underline{5.30 \text{ ft/sec}}}$$

$$U_{sg} = \frac{q_g}{A} = \frac{4}{100} \left[ \frac{q_g}{A} \right] \left[ \frac{\text{ft}^3}{\text{sec}} \right] \left[ \frac{\text{ft}^2}{\text{ft}^2} \right]$$

To calculate the gas flowrate.

$$U_m = U_{se} + U_{sg} = 1.48 + 3.698 = 5.176 \text{ ft/s}$$

Step 3  $\rightarrow$  To get the Baker Flow regime  
to calculate density of oil & gas.  
given that:

$$\gamma_o (\text{s.g.}) = 32^\circ \text{API}$$

$$\gamma_g (\text{s.g.}) = 0.71$$

For oil (---)

$$\text{API} = \frac{141.5}{\text{s.g.}} - 131.5$$

$$32 = \frac{141.5}{\text{s.g.}} - 131.5$$

$$32 + 131.5 = \frac{141.5}{\text{s.g.}}$$

$$163.5 \times \text{s.g.} = 141.5$$

$$\text{s.g.} = \frac{141.5}{163.5} = \underline{\underline{0.865}}$$

Therefore  $\rho_o = 0.865 \times 62.4 = \underline{\underline{53.96 \text{ lbm/ft}^3}}$

For Gas - (---)

Use the Formula: Given  $\gamma_g = 0.71$

$$\rho_g = \frac{2.7 \times \gamma_g \times P}{Z \times T} = \frac{2.7 \times 0.71 \times 1000}{0.85 \times 580}$$

$$\rho_g = \underline{\underline{3.88 \text{ lbm/ft}^3}}$$