

ASSIGNMENT

SECTION A

1 a, charging by Induction:

Electric charges can be obtained on an object without touching it, by a process called electrostatic induction. Consider a negatively charged rubber rod brought near a neutral (uncharged) conducting sphere that is insulated so that there is no conducting path to ground as shown below. The repulsive force between the electrons in the rod and those in the sphere causes a redistribution of charges on the sphere so that some electrons move to the side of the sphere farthest away from the rod ^(fig. 1.3a). The region of the sphere nearest the negatively charged rod has an excess of positive charge because of the migration of electrons away from this location. If a grounded conducting wire is then connected to the sphere, as in ^(fig. 1.3b), some of the electrons leave the sphere and travel to the earth. If the wire to ground is then removed, ^(fig. 1.3c) the conducting sphere is left with an excess of induced positive charge. Finally, when the rubber rod is removed from the vicinity of the sphere

$$F_2 = \frac{kq_2}{r^2} = \frac{9 \times 10^9 \times 8 \times 10^{-6}}{(1.2)^2} = 51397.95918$$

$$F_q = \frac{kq}{r^2} = \frac{9 \times 10^9 \times q}{1} = 9 \times 10^9 q$$

Vector	Angle	X-Component	Y-Component
$F_1 = 51397.95918$	63.4°	$F_1 \cos \theta =$ 2570.046785	$F_1 \sin \theta =$ 5132.262839
$F_2 = 51397.95918$	63.4°	2570.045785	5132.262839
$F_q = 9 \times 10^9 q$	90°	$F_q \cos \theta = 0$ $E_x = 0$	$9 \times 10^9 q$ $E_y = 10264.52568$

$$\text{Magnitude} = \sqrt{(E_x)^2 + (E_y)^2}$$

$$E_q = \sqrt{(0)^2 + (10264.52568)^2}$$

$$\text{Since } E_x = 0$$

$$0 = 9 \times 10^9 q + 10264.52568$$

Making q subject of formula

$$q = \frac{10264.52568}{9 \times 10^9}$$

$$q = 1.140502853 \times 10^{-6}$$

$$q = 1.14 \mu\text{C}$$

$$V_B - V_A = \frac{h}{q_0} (A \cdot n \cdot B)_{\text{avg}} \quad \dots (5)$$

Putting equation (4) in (5) yields $V_B - V_A = - \int_A^B E \cdot dl \quad \dots (6)$

SECTION B

A Magnetic flux is defined as the strength of the magnetic field which can be represented by line of forces. It is represented by the symbol Φ . Mathematically given as $\Phi = \int B \cdot dA$

4b. $m = 9 \times 10^{-31} \text{ kg}$
 $r = 1.4 \times 10^{-7} \text{ m}$

$B = 3.5 \times 10^{-1} \text{ weber/meter}^2$

Cyclotron frequency = angular speed

$$\omega = \frac{v}{r} = \frac{qB}{m}$$

$$\omega = \frac{qB}{m} = \frac{1.6 \times 10^{-19} \times 3.5 \times 10^{-1}}{9 \times 10^{-31}}$$

$$\omega = 6.22 \times 10^{10} \text{ T}^{-1}$$

4c. mass of electron = $9.11 \times 10^{-31} \text{ kg}$

$$4 = 9 \times 10^9 \times 5 \times 10^{-5} q_1 + 9 \times 10^9 q_2$$

$$4 = 4.5 \times 10^5 q_1 + 9 \times 10^9 q_2$$

Quadratic equation

$$9 \times 10^9 q_2 - 4.5 \times 10^5 q_1 + 4 = 0$$

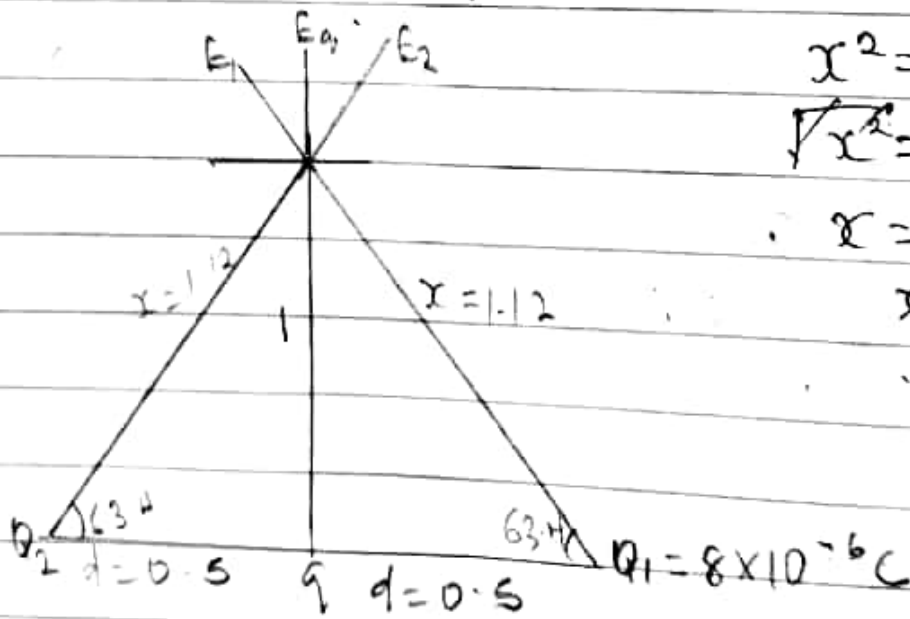
$$q_1 = 0.000011 \text{ C} \approx 1.1 \times 10^{-5} \text{ C}$$

$$q_2 = 0.000038 \text{ C} \approx 3.8 \times 10^{-5} \text{ C}$$

1c. $Q_1 = Q_2 = 8 \mu\text{C}$

$d = 0.5 \text{ m}$

Q. If electric field at a point P is zero.



$$x^2 = 1^2 + 0.5^2$$

$$\sqrt{x^2} = \sqrt{1.25}$$

$$\therefore x = \sqrt{1.25}$$

$$x = 1.12 \text{ m}$$

$$\tan \theta = \frac{\text{Opp}}{\text{Adj}}$$

$$\tan \theta = \frac{1}{0.5}$$

$$\theta = \tan^{-1}(2)$$

$$\theta = 63.4^\circ$$

$$E_1 = \frac{kq_1}{r^2} = \frac{9 \times 10^9 \times 8 \times 10^{-6}}{(1.12)^2} = 57397.9598$$

Recall $dl = dy$

$$B = \frac{\mu_0 I}{4\pi} \int_{-a}^a \frac{x}{(x^2 + y^2)^{3/2}} dy$$

$$B = \frac{\mu_0 I x}{4\pi} \int_{-a}^a \frac{1}{(x^2 + y^2)^{3/2}} dy \quad \text{--- (3)}$$

Using special integrals: $\int \frac{dy}{(x^2 + y^2)^{3/2}} = \frac{1}{x^2} \cdot \frac{y}{(x^2 + y^2)^{1/2}}$

Equation (3) becomes $B = \frac{\mu_0 I x}{4\pi} \left[\frac{y}{x^2 (x^2 + y^2)^{1/2}} \right]_{-a}^a$

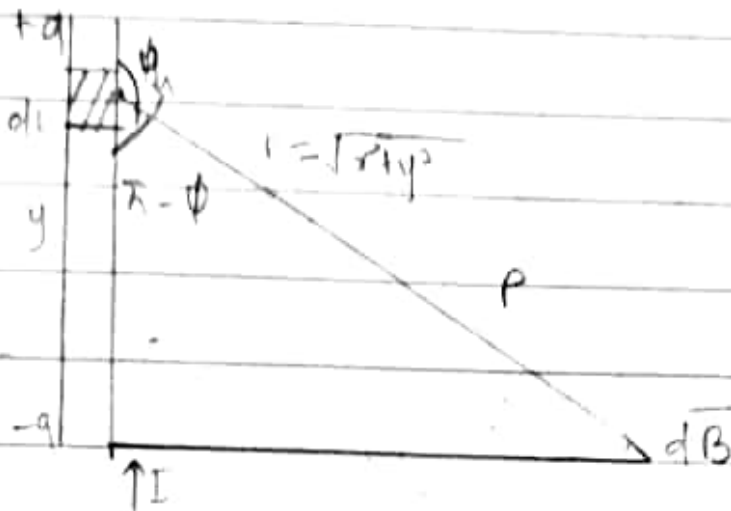
$$B = \frac{\mu_0 I x}{4\pi} \left(\frac{2a}{x^2 (x^2 + a^2)^{1/2}} \right)$$

$$B = \frac{\mu_0 I}{4\pi x} \left(\frac{2a}{\sqrt{x^2 + a^2}} \right)$$

When the length $2a$ of the conductor is very great in comparison to its distance x from point P , we consider it infinitely long. That is, when a is much larger than x , $(x^2 + a^2)^{1/2} \approx a$, as $a \rightarrow \infty$

$$\therefore B = \frac{\mu_0 I}{2\pi x}$$

- In a physical situation, we have axial symmetry about the y -axis. Thus at all points in a circle of radius r , around the conductor, the magnitude of B is: $B = \frac{\mu_0 I}{2\pi r}$ --- # (magnitude of the magnetic field of flux density B near a long straight current carrying conductor)



A section of a straight current carrying conductor

Applying the Biot-Savart law, we find the magnitude of the field dB

$$B = \frac{\mu_0 I}{4\pi} \int_{-a}^{+a} \frac{dl \sin \phi}{r^2}$$

$$\sin(\pi - \phi) = \sin \phi$$

$$\therefore B = \frac{\mu_0 I}{4\pi} \int_{-a}^{+a} \frac{dl \sin(\pi - \phi)}{r^2}$$

From diagram, $r^2 = x^2 + y^2$ (Pythagoras theorem) :

$$B = \frac{\mu_0 I}{4\pi} \int_{-a}^{+a} \frac{dl \sin(\pi - \phi)}{x^2 + y^2} \quad \dots (1)$$

$$\text{But } \sin(\pi - \phi) = \frac{x}{\sqrt{x^2 + y^2}} = \frac{x}{(x^2 + y^2)^{1/2}} \quad \dots (2)$$

$$\text{Substituting (2) into (1), } B = \frac{\mu_0 I}{4\pi} \int_{-a}^{+a} dl \frac{x}{(x^2 + y^2)(x^2 + y^2)^{1/2}}$$

$$B = \frac{\mu_0 I}{4\pi} \int_{-a}^{+a} dl \frac{x}{(x^2 + y^2)^{3/2}}$$

39; Volume charge density, $\rho = \frac{dq}{dV}$ in $dQ = \rho dV$

ii; Surface charge density, $\sigma = \frac{dq}{dA}$ in $dQ = \sigma dA$

iii; Linear charge density, $\lambda = \frac{dq}{dL}$ in $dQ = \lambda dL$

36; Electric potential difference

The electric potential difference between two points in an electric field can be defined as the work done per unit charge against electrical forces when a charge is transported from one point to another. It is measured in Volt (V) or Joules per coulomb (J/C). It is a scalar quantity.

Elemental work done dW is given as:

$$dW = F \cdot dL \quad \text{--- (1)}$$

But $F = -q_0 E$ --- (2)

substituting equation (2) in (1) $= dW = -q_0 E dL$ --- (3)

Total work done in moving the test charge from A to B is:

$$W(A \rightarrow B)_{q_0} = -q_0 \int_A^B E dL \quad \text{--- (4)}$$

From the definition of electric potential difference, it follows that:

$$\text{radius} = 1.4 \times 10^{-7} \text{ m}$$

$$\text{magnetic field} = 3.5 \times 10^{-1} \text{ weber/meter}^2$$

cyclotron frequency can be called Angular speed

$$\text{Recall that Angular speed } \omega = \frac{v}{r} = \frac{qB}{m}$$

$$\text{Substituting we have } \omega = \frac{v}{r} = \frac{qB}{m} = \frac{1.6 \times 10^{-19} \times 3.5 \times 10^{-1}}{9.11 \times 10^{-31}} \\ = 6.22 \times 10^{10} \text{ T}^{-1}$$

So cyclotron frequency = $6.22 \times 10^{10} \text{ T}^{-1}$, the unit is equal to unit of frequency dimensionally.

5. Biot-Savart law states that the magnetic field is directly proportional to the product permeability of free space (μ_0), the current (I), the change in length, the radius and inversely proportional to the square of radius (r^2). It can be represented mathematically by:

$$dB = \frac{\mu_0 I dl \times r}{4\pi r^2} \text{ where } \mu_0 \text{ is a constant called Permeability of free space}$$

$$\mu_0 = 4\pi \times 10^{-7} \text{ T} \cdot \frac{\text{m}}{\text{A}}$$

Unit of B is weber/metre square.

5b, Magnetic Field of a straight current carrying conductor

(Fig 1.31) the induced positive charge remains on the ungrounded sphere and becomes uniformly distributed over the surface of the sphere.

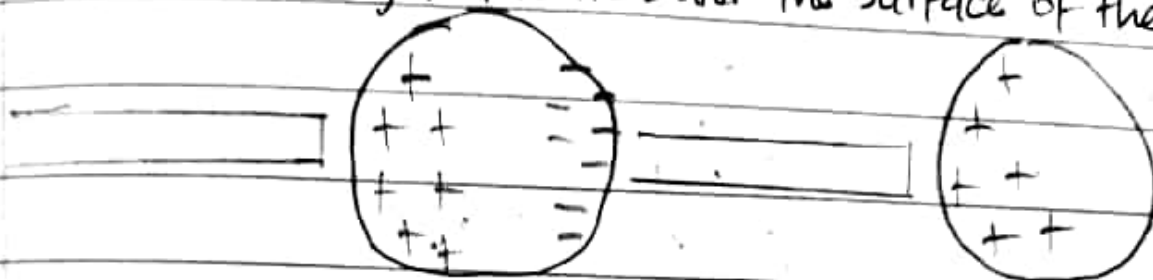


Fig 1.3a

Fig 1.3c

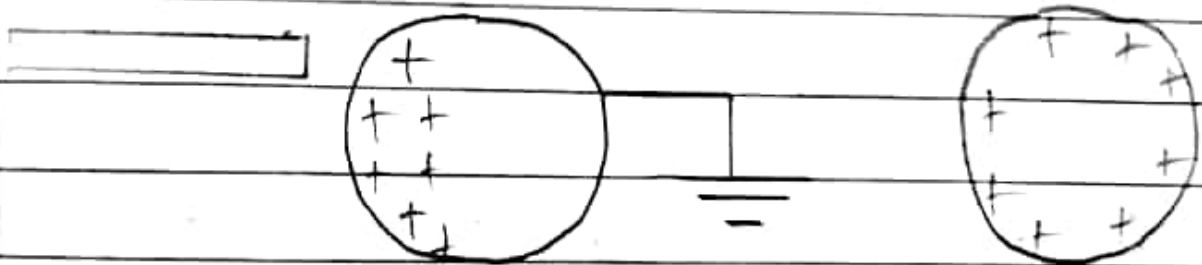


Fig 1.3b

Fig 1.3d

$$1b, \quad k = 9 \times 10^9$$

$$q_1 + q_2 = 5 \times 10^{-5} \text{ C}$$

$$F = 1 \text{ N}$$

$$d = 2 \text{ m}$$

Charge on each sphere = ?

$$F = \frac{k q_1 q_2}{r^2}$$

$$1 = \frac{9 \times 10^9 \times (q_1 q_2 \cdot 5 \times 10^{-5})}{2^2}$$