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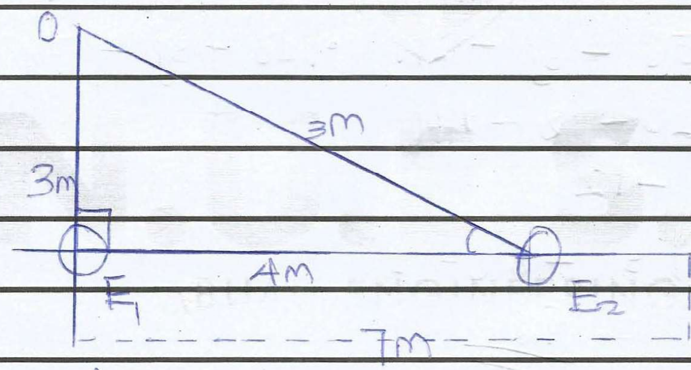
2.a) Electric Field is a region of space in which an electric charge will experience an electric force. While Electric field strength (intensity) ε, can be defined as the force per unit charge. Mathimatically the magnitude of the field, is given by:

$$\varepsilon = \frac{F(N)}{q_0[C]}$$

It is measured in Neewton per ~~colt~~ coulomb [N/C]. The direction of the electric field intensity t, at a point in space is the same as the direction of the force a positive test charge would experience if it is placed at that point.

b)



$$Hyp^2 = Adj^2 + Opp^2$$

$$Hyp^2 = 3^2 + 4^2$$

$$= 9 + 16$$

$$= 25$$

$$= \sqrt{25} = 5m$$

$$\theta = \cos^{-1} 3/4 = 36.9^\circ$$

$$E_1 = \frac{KQ_1}{r^2} = \frac{9\times10^9 \times 8\times10^{-9}}{(7)^2} = 1.469\ N/C$$

$$E_2 = \frac{KQ_2}{r^2} = \frac{9\times10^9 \times 12\times10^{-9}}{(3)^2} = 12\ N/C$$

2(b)

$E_1 + E_2 = 13.469\ N/c$

$\simeq 13.5 N/C$

θ	E	$E_x = E\cos\theta$	$E_y = E\sin\theta$
90°	8	$8\cos 90^\circ$ $= 0$	$8\sin 90^\circ = 8$
36.9°	4.32	$4.32\cos 36.9$ $= -3.455$	$4.32\sin 36.9^\circ = 2.594$ ~~10.594~~
		$= -3.455$	$= 10.594$

$E = \frac{9\times10^9 \times 8\times10^{-9}}{3^2} = 8 N/C$

$E_2 = \frac{9\times10^9 \times 12\times10^{-9}}{5^2} = 4.32 N/C$

$= \sqrt{E_x^2 + E_y^2}$

$= \sqrt{(-3.455)^2 + (10.574)^2}$

$= 11.1 NC$

$\theta = \tan^{-1}\left[\frac{y}{x}\right]$

$\theta = \tan^{-1} \frac{10.574}{-3.455}$

$= 71.9^\circ \simeq 72^\circ$

~~3a)~~

~~(i) Volume charge density, $\rho = \frac{dQ}{dV} \rightarrow dQ = \rho dV$~~

~~(ii) Surface charge density, σ~~

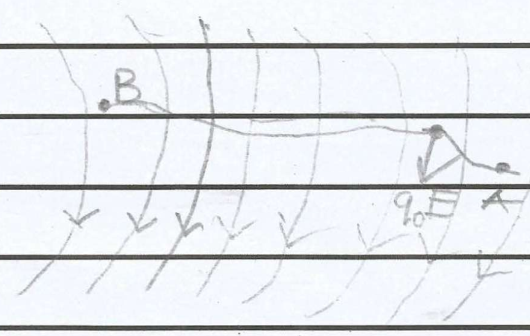
3a)

i) Volume charge density, $\rho = \frac{dQ}{dV} \rightarrow dQ = \rho dV$

ii) Surface charge density, $\sigma = \frac{dQ}{dA} \rightarrow dQ = \sigma dA$

iii) ~~\$~~ Linear charge density, $\lambda = \frac{dQ}{dL} \longrightarrow dQ = \lambda dL$

b) The electric potential difference between two points in an electric field can be defined as the work done per unit charge against electrical forces when a charge is transported from one point to the other. It is measured in Volt (V) or Joules per ~~Colu~~ Coulomb (J/C). Electric potential difference is a scalar quantity



Consider the diagram above suppose a test charge q_0 is moved ~~fr~~ from point A to point B along an arbitrary path inside an electric field E. The electric field E exerts a force $F = q_0E$ on the charge as shown in the diagram above. To move the test charge from point ~~A~~ to point B at constant velocity, an external force of $F = -q_0E$ must act on the

3b)

charge.

~~Th~~ From the definition of ~~electrical~~ potential difference it follows that:

$$V_B - V_A = \frac{W(A \to B)_{Ag}}{q_0}$$

$$V_B - V_A = -\int_A^B E\,dl$$

Explanation:

The elemental work done dW is given as:

$$dW = F \cdot dL \quad \ldots (1)$$

But

$$F = -q_0 E \quad \ldots (2)$$

substituting equation (2) in (1) yields

$$dW = -q_0 E\,dl \quad \ldots (3)$$

Then the total work done in moving the test charge from A to B is:

$$V_B - V_A = \frac{W(A \to B)_{Ag}}{q_0} \quad \ldots (4)$$

Putting eqn (4) in (5)

Cont.

3b.) $W(A \to B)_{Ag} = -q_0 \int_A^B E\,dL$ . . . (4)

From the definition of electric potential difference:

$V_B - U_A = \frac{W(A \to B)_{Ag}}{q_0}$ . . . (5)

putting (4) in (5) yields

$V_B - V_A = -\int_A^B E\,dl$ — (6)

The cyclotron frequency $\omega = \frac{qB}{m_p} = \cancel{\frac{qB}{2m}}$ $\cancel{\frac{qB}{2\pi m}}$

$m = 9.11 \times 10^{-31}$ Kg

$q = 1.6 \times 10^{-19}$ C

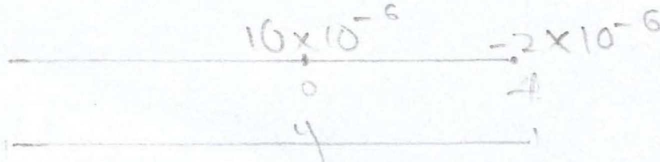
$B = 3.5 \times 10^{-1}$ Weber/meter square

$\omega = \frac{(1.6 \times 10^{-19}) \times (3.5 \times 10^{-1})}{\cancel{2}\,(9.11 \times 10^{-31})}$

$= \cancel{3.074 \times 10^{10}}$ $\cancel{6.147 \times 10^{10}\ rad/s}$

$= 6.147 \times 10^{10}$ rad/s

3c)



$y = x + 4$

$r_1 = x \qquad r_2 = x+4$

potential difference is $\frac{kQ}{r}$

$V_B - V_4 = k\left[\frac{Q_1}{r_1} + \frac{Q_2}{r_2}\right]$

$k\left[\frac{10}{x} + \left[\frac{-2}{x+4}\right]\right]$

At $v = 0$

$k\left[\frac{10(x+4) - 2x}{x(x+4)}\right] = 0$

$k[10(x+4) - 2x] = 0$

$10(x+4) - 2x = 0$

$10x + 40 - 2x = 0$

$10x - 2x = -40$

$x = \frac{-40}{8}$

$x = \cancel{-5\times10^{-6}\,m}$

$x = -5\,m$

4)

a) The magnetic flux (often denoted by ϕ_B) through a surface is the surface integral of the normal component of the magnetic field flux density B passing through that surface. The S.I unit of magnetic flux is the weber (Wb; in derived unit Volts-seconds

It is written mathematically as; $\Phi = BA\cos\theta$

A = test ~~vecta~~ Area , B = magnetic field vector

b) The cyclon frequency $\omega = \frac{qB}{m_e}$

$m_e = 9.11 \times 10^{-31}$ Kg

$q = 1.6 \times 10^{-19}$ C

$B = 3.5 \times 10^{-1}$ web/Meter square

$$\omega = \frac{(1.6 \times 10^{-19}) \times (3.5 \times 10^{-1})}{(9.11 \times 10^{-31})}$$

$= 6.147 \times 10^{10}$ rad/s

c) This is because the charge particle moves in circular motion at this angular frequency in a machine called the cyclotron.

5) a) The Biot-Savart Law is based on the following observations for the magnetic field $d\vec{B}$ at a point P associated with a ~~test~~ length element $d\hat{l}$ of a wire carrying a steady current I:

Observations from the Biot-Savart Experiment.

1) The vector $d\vec{B}$ is pependicular both to $d\hat{l}$ (which points in the direction of the current) and to the unit vector $\hat{r}$ directed from $d\vec{l}$ towards P.

2) The magnitude of $d\vec{B}$ is inversly proportional to r^2, where r is the distance from $d\vec{l}$ to P

3) The magnitude of $d\vec{B}$ is proportional to the current I and to the magnitude of the length element $d\vec{l}$.

4) The magnitude of $d\vec{B}$ is proportional to $n\theta$, where θ is the angle between $\hat{r}$ and $d\vec{l}$.

These observations are summarized in the mathematical expression known as Biot-Savart Law:

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I\, d\vec{l} \times \hat{r}}{r^2} \qquad (1)$$

where μ_0 is a constant called Permiability of free space

$$\mu_0 = 4\pi \times 10^{-7}\ T.m/A$$

Note: the magnetic field $d\vec{B}$ defined in equation (1) is the field created at some point by the current in only a small length $d\vec{l}$ of the conductor.

Therefore, the total magnetic field $\vec{B}$ created at some point by a current I of the finite size is given by integrating equation (1), thus;

$$\vec{B} = \frac{\mu_0}{4\pi}\int \frac{I\, d\vec{l} \times \hat{r}}{r^2} \qquad (2)$$

The magnitude of the magnetic field is given as

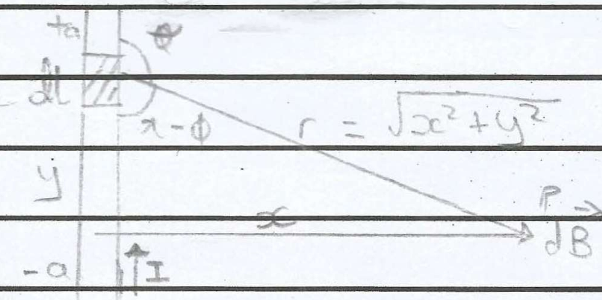
$$B = \frac{\mu_0}{4\pi}\int \frac{I\, dl \sin\theta}{r^2} \qquad (3)$$

5b) From the Biot-savart law, we find the magnitude of the field $d\vec{B}$

$$B = \frac{\mu_0 I}{4\pi}\int_{-a}^{a} \frac{dl \sin\phi}{r^2}$$

$$\sin(\pi - \phi) = \sin\theta$$

$$\therefore B = \frac{\mu_0 I}{4\pi}\int_{-a}^{a} \frac{dl \sin(\pi - \phi)}{r^2}$$



From diagram $r^2 = x^2 + y^2$. (Pythagoras theorem)

$$B = \frac{\mu_0 I}{4\pi}\int_{-a}^{a} \frac{dl \sin(\pi - \phi)}{x^2 + y^2} \quad \text{- - - } (*)$$

φ

5b) but $\sin(\pi - \phi) = \frac{x}{\sqrt{x^2+y^2}} = \frac{x}{(x^2+y^2)^{1/2}}$ (**)

Substituting (**) into (*), we have

$$B = \frac{\mu_0 I}{4\pi}\int_{-a}^{a} dl \frac{x}{(x^2+y^2)(x^2+y^2)^{1/2}}$$

$$B = \frac{\mu_0 I}{4\pi}\int_{-a}^{a} dl \frac{x}{(x^2+y^2)^{3/2}}$$

Recall $dl = dy$

$$B = \frac{\mu_0 I}{4\pi}\int_{-a}^{a} \frac{x}{(x^2+y^2)^{3/2}} dy$$

$$B = \frac{\mu_0 I x}{4\pi}\int_{-a}^{a} \frac{1}{(x^2+y^2)^{3/2}} dy \quad (***)$$

Using special integrals:

$$\int \frac{dy}{(x^2+y^2)^{3/2}} = \frac{1}{x^2}\frac{y}{(x^2+y^2)^{1/2}}$$

Equation (***) therefore becomes

$$B = \frac{\mu_0 I x}{4\pi}\left[\frac{y}{x^2(x^2+y^2)^{1/2}}\right]_{-a}^{a}$$

$$B = \frac{\mu_0 I x}{4\pi}\left(\frac{2a}{x^2(x^2+a^2)^{1/2}}\right)$$

$$B = \frac{\mu_0 I}{4\pi x}\left(\frac{2a}{(x^2+a^2)^{1/2}}\right)$$

When the length $2a$ of the conductor is very great in comparison to its distance x from point P, we consider it infinitely long. That is, when a is much larger than x. If a is far greater than x, x is considered negligible.