

① A particle moves along a curve, $x = 4t^3$, $y = 5t^2$, $z = t + 7$, where t is time. Find its acceleration

$$r = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$$

$$r = t^2\mathbf{i} + 5t^2\mathbf{j} + (t + 7)\mathbf{k}$$

$$\frac{dr}{dt} = 2t\mathbf{i} + 10t\mathbf{j} + \mathbf{k}$$

$$\frac{d^2r}{dt^2} = 2\mathbf{i} + 10\mathbf{j}$$

② If $P = \mathbf{i} - 9\mathbf{j} - 4\mathbf{k}$, $Q = 8\mathbf{i} + 3\mathbf{j} + 6\mathbf{k}$, $R = \mathbf{i} - 7\mathbf{j} - 3\mathbf{k}$
Find $(P \times Q) \cdot (R \times P)$

$$P \times Q = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -9 & -4 \\ 8 & 3 & 6 \end{vmatrix} = \mathbf{i} \begin{vmatrix} -9 & -4 \\ 3 & 6 \end{vmatrix} - \mathbf{j} \begin{vmatrix} 1 & -4 \\ 8 & 6 \end{vmatrix} + \mathbf{k} \begin{vmatrix} 1 & -9 \\ 8 & 3 \end{vmatrix}$$

$$= \mathbf{i}(-54 + 12) - \mathbf{j}(6 + 32) + \mathbf{k}(3 + 72)$$

$$= -42\mathbf{i} - 38\mathbf{j} + 75\mathbf{k}$$

$$R \times P = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -4 & -3 \\ 1 & -9 & -4 \end{vmatrix} = \mathbf{i} \begin{vmatrix} -4 & -3 \\ -9 & -4 \end{vmatrix} + \mathbf{j} \begin{vmatrix} 1 & -3 \\ 1 & -4 \end{vmatrix} + \mathbf{k} \begin{vmatrix} 1 & -4 \\ 1 & -9 \end{vmatrix}$$

$$= \mathbf{i}(16 - 27) - \mathbf{j}(-4 + 3) + \mathbf{k}(-9 + 4)$$

$$= -11\mathbf{i} + \mathbf{j} - 5\mathbf{k}$$

$$\begin{aligned}
 (P \times Q) \cdot (R \times P) &= -42 \times -11 - 38 \times 1 + 75 \times -5 \\
 &= 462 - 38 - 375 \\
 &= 49
 \end{aligned}$$

③ Given $F = 5 \cos 7t \mathbf{i} - 2e^{3t} \mathbf{j} - 4t^3 \mathbf{k}$, find the integral of F with respect to t

$$\int F = 5 \int \cos 7t \mathbf{i} dt - 2 \int e^{3t} \mathbf{j} dt - 4 \int t^3 \mathbf{k} dt$$

$$\begin{aligned}
 \frac{dy}{dt} &= 7 \\
 dt &= \frac{dy}{7}
 \end{aligned}$$

$$\therefore \int F = 5 \int \frac{\cos u}{7} \mathbf{i} du - 2 \int e^{3+t} \mathbf{j} dt - 4 \int t^3 \mathbf{k} dt$$

$$\int F = \frac{5}{7} \int \sin u \mathbf{i} - \frac{2}{3} e^3 - t^4 + C$$