17/ENG 05/081

MAT 104 MECHATRONICS ENGINEERING

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(1) $\int x^2 \sin x \, dx$

Integrate by parts: $\int f g' = fg - \int f' g$

$f = x^2, \quad g' = \sin x$

$f' = 2x \quad g = -\cos x$

$= -x^2 \cos x - \int -2x \cos x \, dx$

$= -x^2 \cos x + 2\int x \cos x \, dx$

$f = x \quad g' = \cos(x)$

$f' = 1 \quad g = \sin(x)$

$= -x^2 \cos x + 2(x \sin x - \int \sin(x) dx)$

$= -x^2 \cos x + 2(x \sin x + \cos x)$

$= 2x \sin x - x^2 \cos x + 2\cos x + c$

$= 2x \sin x + (2 - x^2)\cos x + c$

(2) $\int 3te^{2t} dt$

$3\int te^{2t} dt$

$f = t \quad g' = e^{2t}$

$f' = 1 \quad g = \frac{e^{2t}}{2}$

$= \frac{te^{2t}}{2} - \int \frac{e^{2t}}{2} dt$

$\int \frac{e^{t}}{2} dt \quad u = 2t \quad \frac{du}{dt} = 2 \quad dt = \frac{1}{2} du$

$\frac{1}{4} \int e^u du$ ~~$\frac{e^u}{}$~~

$\int a^u du = \frac{a^u}{\ln a}$ with $a = e$

$\frac{1}{4} \int e^u du$

$= \frac{e^u}{4} = \frac{e^{2t}}{4}$

$\frac{te^{2t}}{2} - \frac{e^{2t}}{4}$

$= 3\left(\frac{te^{2t}}{2} - \frac{e^{2t}}{4}\right)$

$=$ ~~$\frac{3}{4}(2t-1)e^{2t} + c$~~ $\frac{3}{4}(2t-1)e^{2t} + c$

(3) $\int 2x^2 \ln(x)\, dx$

$2 \int x^2 \ln(x)\, dx$

$\int f g' = fg - \int f' g$

$f = \ln(x) \quad g' = x^2 \quad f' = \frac{1}{x} \quad g = \frac{x^3}{3}$

$= 2\left(\frac{x^3 \ln(x)}{3} - \int \frac{x^2}{3} dx \right)$

$= 2\left(\frac{x^3 \ln x}{3} - \frac{1}{3} \int x^2 dx \right)$

$= 2\left(\frac{x^3 \ln x}{3} - \frac{x^3}{3} \cdot \frac{1}{3} \right)$

$= 2\left(\frac{x^3 \ln x}{3} - \frac{x^3}{9} \right)$

$= \frac{2x^3(3\ln x - 1)}{9} + c$

(4) $\int \frac{2x - 3x^2}{1 - x}$

$= \int \frac{x(3x-2)}{x-1} dx$

$u = x - 1 \quad \frac{du}{dx} = 1 \quad dx = du$

$= \int \frac{(u+1)(3u+1)}{u} du$

$= \int (3u + \frac{1}{u} + 4) du$

$= 3\int u\, du + \int \frac{1}{u} du + 4 \int 1\, du$

$= \frac{3u^2}{2} + \ln u + 4u$

$= \frac{3(x-1)^2}{2} + \ln(x-1) + 4(x-1)$

$= \frac{3x^2}{2} + x + \ln(x-1) + c$