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- 1) A particle moves along a curve,  $x = t^3$ ,  $y = 5t^2 + t$ ,  $z = t + 7$ , where  $t$  is time. Find its acceleration

Solution

$$\text{Acceleration} = \frac{d^2r}{dt^2}$$

To find the acceleration, we must first find  $dr/dt$

$$r = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$$

$$r = (t^3)\mathbf{i} + (5t^2 + t)\mathbf{j} + (t + 7)\mathbf{k}$$

$$\frac{dr}{dt} = (2t)\mathbf{i} + (10t + 1)\mathbf{j} + (1)\mathbf{k}$$

$$\frac{dr}{dt} = 2\mathbf{i} + 10\mathbf{j} + \mathbf{k} \quad \therefore \frac{d^2r}{dt^2} = 2\mathbf{i} + 10\mathbf{j} + \mathbf{k}$$

2. If  $P = \mathbf{i} - 9\mathbf{j} - 4\mathbf{k}$ ,  $Q = 8\mathbf{i} - 3\mathbf{j} + 6\mathbf{k}$ ,  $R = \mathbf{i} - 4\mathbf{j} - 3\mathbf{k}$ , Find:  
 $(\vec{P} \times \vec{Q}) \cdot (\vec{R} \times \vec{P})$

$$\vec{P} \times \vec{Q} = (\mathbf{i} - 9\mathbf{j} - 4\mathbf{k}) \times (8\mathbf{i} - 3\mathbf{j} + 6\mathbf{k})$$

$$(\vec{P} \times \vec{Q}) = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -9 & -4 \\ 8 & -3 & 6 \end{vmatrix}$$

$$= \mathbf{i} \begin{vmatrix} -9 & -4 \\ -3 & 6 \end{vmatrix} - \mathbf{j} \begin{vmatrix} 1 & -4 \\ 8 & 6 \end{vmatrix} + \mathbf{k} \begin{vmatrix} 1 & -9 \\ 8 & -3 \end{vmatrix}$$

$$= \mathbf{i}(-54 + 12) - \mathbf{j}(6 - (-32)) + \mathbf{k}(-3 - (-72))$$

$$= -42\mathbf{i} - 38\mathbf{j} + 69\mathbf{k}$$

$$(\vec{R} \times \vec{P}) = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -4 & -3 \\ 1 & -9 & -4 \end{vmatrix}$$

$$\mathbf{i} \begin{vmatrix} -4 & -3 \\ -9 & -4 \end{vmatrix} - \mathbf{j} \begin{vmatrix} 1 & -3 \\ 1 & -4 \end{vmatrix} + \mathbf{k} \begin{vmatrix} 1 & -4 \\ 1 & -9 \end{vmatrix}$$

$$\mathbf{i}(16 - 27) - \mathbf{j}(-4 - (-3)) + \mathbf{k}(-9 - (-4))$$

$$= -11\mathbf{i} + \mathbf{j} - 5\mathbf{k}$$

$$\begin{aligned}
 (\vec{R} \times \vec{Q}) \cdot (\vec{R} \times \vec{P}) &= (-42\vec{i} - 38\vec{j} + 69\vec{k}) \cdot (-11\vec{i} + \vec{j} - 5\vec{k}) \\
 &= 462 - 38 - 345 \\
 &= 79
 \end{aligned}$$

3. Given  $F = 5 \cos 7t \vec{i} - 2e^{3t} \vec{j} - 4t^3 \vec{k}$ , find the integral of  $F$  with respect to  $t$

Solution

$$\int F dt = \int 5 \cos(7t) \vec{i} dt - \int 2e^{3t} \vec{j} dt - \int 4t^3 \vec{k} dt$$

$$= \frac{5 \sin(7t)}{7} - \frac{2e^{3t}}{3} - \frac{4t^4}{4} + C$$

$$\int F dt = \frac{5 \sin(7t)}{7} - \frac{2e^{3t}}{3} - t^4 + C$$