

1. $\int \frac{2x}{\sqrt{4x^2-1}} dx$

Sol

Let $u = 4x^2 - 1$

$\left(\frac{u-1}{4}\right)^{\frac{1}{2}} = x$

$x = \left(\frac{u-1}{4}\right)^{\frac{1}{2}} \Rightarrow \frac{(u-1)^{\frac{1}{2}}}{2}$

$\frac{dx}{du} = \frac{1}{4(u-1)^{\frac{1}{2}}}$

$dx = \frac{du}{4(u-1)^{\frac{1}{2}}}$

$\therefore \int \frac{2x}{\sqrt{4x^2-1}} dx = \int \frac{2\left(\frac{u-1}{4}\right)^{\frac{1}{2}} \times \frac{1}{4(u-1)^{\frac{1}{2}}} \times du}{u^{\frac{1}{2}}}$

$\int \frac{2x}{\sqrt{4x^2-1}} dx = \frac{1}{4} \int u^{-\frac{1}{2}} du$

$\int \frac{2x}{\sqrt{4x^2-1}} dx = \frac{1}{4} \left[\frac{u^{\frac{1}{2}}}{\frac{1}{2}} \right] + C$

$\int \frac{2x}{\sqrt{4x^2-1}} dx = \frac{2}{4} \left[(4x^2-1)^{\frac{1}{2}} \right] + C$

$\int \frac{2x}{\sqrt{4x^2-1}} dx = \frac{1}{2} \left[\sqrt{4x^2-1} \right] + C$

$\int \frac{\sin^{-1}x}{\sqrt{1-x^2}} dx = u du$

$\int \frac{\sin^{-1}x}{\sqrt{1-x^2}} dx = \frac{u^2}{2} + C$

$\int \frac{\sin^{-1}x}{\sqrt{1-x^2}} dx = \frac{(\sin^{-1}x)^2}{2} + C$

3) $\int (\tan x)^6 \sec^2 x dx$

Sol

$u = \tan x, \frac{du}{dx} = \sec^2 x$

$dx = \frac{du}{\sec^2 x}$

$= \int u^6 \times \sec^2 x \times \frac{du}{\sec^2 x}$

$= \int du$

$\int (\tan x)^6 \sec^2 x dx = \frac{u^7}{7} + C$

$\int (\tan x)^6 \sec^2 x dx = \frac{(\tan x)^7}{7} + C$

2. $\int \frac{\sin^{-1}x}{\sqrt{1-x^2}} dx$

Let $u = \sin^{-1}x$

$\frac{du}{dx} = \frac{1}{\sqrt{1-x^2}}$

$dx = (\sqrt{1-x^2}) du$

$\int \frac{\sin^{-1}x}{\sqrt{1-x^2}} dx = \int u \times \sqrt{1-x^2} du$