

19/ENG05/017

A particle moves along a curve,  $x = t^2$ ,  $y = -5t^2 + t$ ,  $z = t + 7$ , where  $t$  is time. Find its acceleration.

solution

$$\vec{r} = xi + yj + zk$$

$$\text{velocity} = \frac{d\vec{r}}{dt} \quad \vec{r} = t^2i + (-5t^2 + t)j + (t + 7)k$$

$$\frac{d\vec{r}}{dt} = \cancel{(2t)i + (-10t + 1)j + 1k} \quad 2ti + (-10t + 1)j + 1k$$

$$\text{Acceleration} = \frac{d^2\vec{r}}{dt^2}$$

$$\frac{d^2\vec{r}}{dt^2} = 2i - 10j$$

$$\therefore \text{Acceleration} = 2i - 10j$$

If  $P = i - 9j - 4k$ ,  $Q = 8i - 3j + 6k$ ,  $R = i - 4j - 3k$ , find  $(P \times Q)$ ,  $(R \times P)$

Solution

$$P \times Q = \begin{vmatrix} i & j & k \\ 1 & -9 & -4 \\ 8 & -3 & 6 \end{vmatrix}$$

$$P \times Q = [(-9 \times 6) - (-4 \times -3)]i - [(1 \times 6) - (-4 \times 8)]j + [(1 \times -3) - (-9 \times 8)]k$$

$$(-54 - 12)i - (6 + 32)j + (-3 + 72)k$$

$$-66i - 38j + 69k$$

$$R \times P = \begin{vmatrix} i & j & k \\ 1 & -4 & -3 \\ 1 & -9 & -4 \end{vmatrix}$$

$$R \times P = [(-4 \times -4) - (-3 \times -9)]i - [(1 \times -4) - (-3 \times 1)]j + [(1 \times -9) - (-4 \times 1)]k$$

$$(16 - 27)i - (-4 + 3)j + (-9 + 4)k$$

$$\cancel{11i + 7j} - 5k \quad -11i + j - 5k$$

$$(P \times Q) \cdot (R \times P)$$

$$P \times Q = -66i - 38j + 69k$$

$$R \times P = -11i + 7j - 5k$$

$$(-66i - 38j + 69k) \cdot (-11i + 7j - 5k)$$

$$(P \times Q) \cdot (R \times P) = 726 - 38 \times 7 - 345$$

$$(P \times Q) \cdot (R \times P) = 343$$

3. Given  $f = 5 \cos 7ti - 2e^{3t}j - 4t^3k$ , find the integral of  $f$  with respect to  $t$   
solution

$$\int f = \int 5 \cos 7ti - 2e^{3t}j - 4t^3k \, dt$$

$$\frac{5 \times 1 \sin 7ti}{7} - \frac{2 \times 1 e^{3t}j}{3} - \frac{4t^{3+1}k}{3+1}$$

$$\frac{5 \sin 7ti}{7} - \frac{2}{3} e^{3t}j - \frac{4t^4}{4} k + C$$

$$\frac{5 \sin 7ti}{7} - \frac{2}{3} e^{3t}j - t^4 k + C$$