$= -x^2\cos x + 2x\sin x + 2\cos x + C$

3. $\sin 7x \cos 2x$

Solution

$\int \sin 7x \cos 2x$

Recall,

$$\boxed{\sin a \cdot \cos b = \frac{1}{2}\left[\sin(a+b) + \sin(a-b)\right]}$$

$$\sin(7x)\cos(2x) = \frac{1}{2}\left[\sin(9x) + \sin(5x)\right]$$

$$= \frac{1}{2}\int \sin(9x)\,dx + \int \frac{1}{2}\sin(5x)\,dx$$

$$= \frac{1}{2}\left(\frac{-\cos(9x)}{9}\right) + \frac{1}{2}\left(\frac{-\cos(5x)}{5}\right) + C$$

$$= \frac{-\cos(9x)}{18} - \frac{\cos(5x)}{10} + C$$

4. $\frac{(2x - 3x^2)}{1-x}$

Solution

$$\int \frac{2x - 3x^2}{1-x}\,dx =$$

$\cancel{u = 1-x}$

$\cancel{du = -1\,dx}$

$\cancel{du = -dx}$

$\cancel{x = 1-u}$

let $u = 2x - 3x^2$

$du = 2 - 6x\,dx$

$dx = \frac{du}{2-6x}$

$$\int \frac{u}{1-x} \times \frac{du}{2-6x}$$

$$\int \frac{u}{(1-x)(2-6x)}\,du = \int \frac{A}{(1-x)} + \frac{B}{(2-6x)}$$

Multiply through by $(1-x)(2-6x)$

$2x - 3x^2 = (2-6x)A + B(1-x)$

when $x = 1$ ; when $x = 1/3$

$-1 = -4A$ $\quad$ $1/3 = 2/3 B$

$A = 1/4$ $\quad$ $B = 1/2$

$$\int 4(1-x) + \int 2(2-6x)$$

$$-\frac{4}{2}(x) + \frac{-2}{2}(x) + C$$

$$= -2x - (x) + C$$

$$= -3x + C$$

NAME: ISAAC ENE GRACE
DEPARTMENT: BIOMEDICAL ENGINEERING,
MATRIC NUMBER: 19/ENG05/004

MAT 104

Integrate the following with respect to their variable:

1. $3te^{2t}$

Solution

$\int 3te^{2t}\,dt$

$3\int te^{2t}\,dt$

Recall;

$\int u\,dv = uv - \int v\,du$

where;

$u = t$, $dv = e^{2t}$

$du = dt$, $\int dv = \int e^{2t}$

$v = \frac{1}{2}e^{2t}$

Substitute into the equation

$= t \cdot \frac{1}{2}e^{2t} - \int \frac{1}{2}e^{2t} \cdot dt$

$= \frac{1}{2}te^{2t} - \frac{1}{2}\int e^{2t} \cdot dt$

$= \frac{1}{2}te^{2t} - \frac{1}{4}e^{2t} \cancel{+} C$

$3\left(\frac{1}{2}te^{2t} - \frac{1}{4}e^{2t} + c\right) = \frac{3}{2}te^{2t} - \frac{3}{4}e^{2t} + c$

2. $x^2 \sin x$

Solution

$\int x^2 \sin x\,dx = uv - \int v\,du$

where;

$u = x^2$

$du = 2x\,dx$

$dv = \sin x$

$v = -\cos x$

$= -x^2\cos x - \int -\cos x \cdot 2x\,dx$

$= -x^2\cos x + \int 2x\cos x\,dx$

$= -x^2\cos x + 2x\sin x - \int 2\sin x\,dx$

$= -x^2\cos x + 2x\sin x - (-2\cos x)$