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19/NIITSO1/351

1. $\int \frac{1}{\sqrt{4-x^2}}\, dx$

$x = 2\sin\theta$

$\frac{dx}{d\theta} = 2\cos\theta$

$dx = 2\cos\theta\, d\theta$

$a^2 - x^2 = 4 + 4\sin\theta$

$a^2 - x^2 = 4(1+\sin\theta)$

$a^2 - x^2 = 4\cos\theta$

$\int \frac{1}{\sqrt{4-x^2}}\, dx = \int \frac{2\cancel{\cos\theta}\, d\theta}{4\cancel{\cos\theta}}$

$\frac{1}{2}\int d\theta$

$\frac{1}{2}\theta + c$

$\frac{4}{2}$ $x = 2\sin\theta$

$\sin\theta = \frac{x}{2}$

$\theta = \sin^{-1}\left(\frac{x}{2}\right)$

$\therefore \frac{1}{2}\sin^{-1}\frac{x}{2} + C$

$\boxed{\frac{1}{2}\sin^{-1}\frac{x}{2} + c}$

2) $\int \frac{1}{4+x^2}\, dx$

$x = 2\tan\theta$

$\frac{dx}{d\theta} = 2\sec^2\theta$

$dx = 2\sec^2\theta\, d\theta$

$x = 2\tan\theta$

$\theta = \tan^{-1}\left(\frac{x}{2}\right)$

$a^2 + x^2 = 4 + 4\tan\theta$

$a^2 + x^2 = 4(1+\tan\theta)$

$a^2 + x^2 = 4\sec^2\theta$

$\int \frac{1}{4+x^2}\, dx = \int \frac{2\cancel{\sec^2\theta}\, d\theta}{4\cancel{\sec^2\theta}}$

$\frac{1}{2}\int d\theta$

$\frac{1}{2}\theta + c = \frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right) + c$

$= \frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right) + c$

$$\int \frac{1}{9x^2+4}\, dx$$

$9x^2 = 4\tan^2\theta$

$x^2 = \frac{4\tan^2\theta}{9}$

$x = \frac{2\tan\theta}{3}$

$\frac{dx}{d\theta} = \frac{2\sec^2\theta}{3}$

$dx = \frac{2\sec^2\theta \, d\theta}{3}$

$x = \frac{2\tan\theta}{3}$

$\tan\theta = \frac{3x}{2}$

$\theta = \tan^{-1}\left(\frac{3x}{2}\right)$

$4^2 + x^2 = 4 + 4\tan\theta$

$4^2 + x^2 = 4(1 + \tan\theta)$

$4^2 + x^2 = 4\sec^2\theta$

$$\int \frac{1}{9x^2+4}\, dx = \int \frac{2\sec^2\theta \, d\theta}{3} \div \frac{4\sec^2\theta}{1}$$

$$\int \frac{2\sec^2\theta \, d\theta}{3} \times \frac{1}{4\sec^2\theta}$$

$9x^2 = \ldots$

$x^2 = \ldots$

$$\int \frac{2\cancel{\sec^2\theta}\,d\theta}{3} \times \frac{1}{4\cancel{\sec^2\theta}}$$

$$\frac{2}{12}\int d\theta$$

$$\frac{1}{6}\theta + c$$

$$\frac{1}{6}\tan^{-1}\left(\frac{3x}{2}\right) + c$$