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Math 104

Questions

Integrate the following with respect to their Variable.

1. $3te^{2t}$
2. $x^2 \sin x$
3. $\sin 7x \cos 2x$
4. $\dfrac{2x-3x^2}{1-x}$

Solutions.

1. $\int 3te^{2t}$

$u = 3t \qquad v = e^{2t}$

$\dfrac{du}{dt} = 3 \qquad dv = e^{2t}$

$du = 3dt$

$\int u\,dv = uv - \int v\,du$

$= 3t\left(\dfrac{e^{2t}}{2}\right) - \int \dfrac{e^{2t}}{2} \cdot 3dt$

$$= \frac{3te^{2t}}{2} - \frac{3}{2}\int e^{2t}\,dt$$

$$= \frac{3te^{2t}}{2} - \frac{3}{2}\left[\frac{e^{2t}}{2}\right] + C$$

$$\therefore \int 3te^{2t} = \frac{3te^{2t}}{2} - \frac{3e^{2t}}{4} + C$$

2. $\int x^2 \sin x$

$u = x^2 \quad dv = \sin x \quad v = -\cos x$

$du \; 2x\,dx$

$\int u\,dv = uv - \int v\,du$

$= x^2(-\cos x) - \int -\cos x \cdot 2x\,dx$

$= -x^2\cos x - \int -2x\cos x\,dx$

$\int -2x\cos x\,dx$

$u = -2x \quad dv = \cos x \quad v = \sin x$

$dv = -2dx$

$\int u\,dv = uv - \int v\,du$

$= -2x\sin x - \int \sin x \cdot -2dx$

$= -2x\sin x - \int -2\sin x\,dx$

$= +2x\sin x + 2\int \sin x\,dx$

$= [-2x\sin x + 2[-\cos x]]$

$\int -2x\cos x\,dx = -2x\sin x - 2\cos x$

$\therefore \int x^2\sin x = x^2\cos x - (-2x\sin x - 2\cos x)$

$\int x^2\sin x = -x^2\cos x + 2x\sin x + 2\cos x + C$

3 $\int \sin 7x \cos 2x$

$= \frac{1}{2}[\sin 9x + \sin 5x]$

$= \frac{\sin 9x}{2} + \frac{\sin 5x}{2}$

$\therefore \int \frac{\sin 9x}{2} + \int \frac{\sin 5x}{2}$

$\frac{1}{2}\int \sin 9x + \frac{1}{2}\int \sin 5x$

$\frac{1}{2}\left(\frac{-\cos 9x}{9}\right) + \frac{1}{2}\left(\frac{-\cos 5x}{5}\right) + C$

$= -\frac{\cos 9x}{18} - \frac{\cos 5x}{10} + C$

4. $\int \frac{2x - 3x^2}{1 - x}$

$$\begin{array}{r|l} & 3x+1 \\ \hline -x+1 & -3x^2+2x \\ & -3x^2+3x \\ \hline & -x \\ & -x+1 \\ \hline & -1 \end{array}$$

$\int 3x + 1 - \frac{1}{1+x}$

$$\int 3x + \int 1 - \int 1/x - x$$

$$3/2\,x^2 + x - [-\ln(1-x)] + C$$

$$\therefore \int \frac{2x - 3x^2}{1-x} = 3/2\,x^2 + x + \ln(1-x) + C$$