

Date 23rd April, 2020  
 Day Thursday  
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 Course Mat 102  
 Dept Aeronautics  
 College Engineering  
 Matric 19/ENCA09/1008.

- ① Find the equation of the tangent at the point (1,0) on the circle  $x^2 + y^2 - 5x - y + 4 = 0$
- ② Find the equation of the tangent at the point (1,0) on the circle  $x^2 + y^2 - 12x - 17y + 47 = 0$
- ③ Find the equation of the tangent at the point (3,0) on the circle  $x^2 + y^2 - 8x + 11y + 10 = 0$ .

Solution:

①  $x^2 + y^2 - 5x - y + 4 = 0$

Using completing the square method

$x^2 - 5x + y^2 - y = -4$   
 add half the coefficient <sup>squared</sup> of x and y to both sides

$$x^2 - 5x + \frac{25}{4} + y^2 - y + \frac{1}{4} = -4 + \frac{25}{4} + \frac{1}{4}$$

collect the squares

$$\left(x - \frac{5}{2}\right)^2 + \left(y - \frac{1}{2}\right)^2 = \frac{-16 + 25 + 1}{4} = \frac{10}{4}$$

Formula for centre and radius of circle

$$(x-h)^2 + (y-k)^2 = r^2$$

where (h, k) = centre and r = radius

∴ The centre of circle =  $\left(\frac{5}{2}, \frac{1}{2}\right)$  at point (1,0)

and radius =  $\sqrt{10/4} = \frac{\sqrt{10}}{2}$

Find the gradient of P

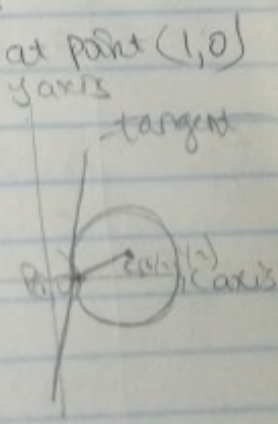
$$m = \frac{y_2 - y_1}{x_2 - x_1} \text{ where } y_2 = \frac{1}{2}, y_1 = 0$$

$$x_2 = \frac{5}{2}, x_1 = 1$$

$$m = \frac{\frac{1}{2} - 0}{\frac{5}{2} - 1} = \frac{\frac{1}{2}}{\frac{5-2}{2}} = \frac{1}{2} \div \frac{3}{2}$$

$$m = \frac{1}{2} \times \frac{2}{3}$$

$$m = \frac{1}{3} = \frac{1}{3}$$



gradient of tangent = -3/11

$$\begin{aligned}\text{Equation of tangent} &= y - y_1 = m(x - x_1) \\ &= y - 0 = -3(x - 1) \\ &= y = -3x + 3\end{aligned}$$

(2)  $x^2 + y^2 - 12x - 12y + 47 = 0$

Using completing the square method.

$$x^2 - 12x + y^2 - 12y + 47 = 0$$

add half the coefficient of  $x$  and  $y$  squared to both side.

$$x^2 - 12x + 36 + y^2 - 12y + 36 = -47 + 36 + 36$$

collect the squares

$$(x - 6)^2 + (y - 6)^2 = 25$$

Using the centre and radius formula which is

$$(x - h)^2 + (y - k)^2 = r^2$$

where centre =  $(h, k)$  and radius =  $r$

$\therefore$  centre =  $(6, 6)$  and radius = 5, at  $P_2(x_1, y_1)$

find the gradient of  $CP$

$$\text{gradient } CP = \frac{y_2 - y_1}{x_2 - x_1} \quad y_2 = 6, y_1 = 0 \\ x_2 = 6, x_1 = 1$$

$$\text{gradient } C-P = \frac{6 - 0}{6 - 1} = \frac{6}{5}$$

$\therefore$  the gradient of tangent

$$m = -\frac{5}{6}$$

The equation  $y - y_1 = m(x - x_1)$

$$y - 0 = m(x - 1)$$

$$y = m(x - 1)$$

$$y = -\frac{5}{6}(x - 1)$$

$$y = -\frac{5}{6}x + \frac{5}{6} \quad \text{multiply by 6}$$

$\therefore$  the equation of  $26y = -5x + 5$  or

$$\text{tangent of circle } 26y + 5x - 5 = 0$$

(3)  $x^2 + y^2 = 25$

using comp

$$x^2 - 8x + 4$$

add half

of 8

$$x^2 - 8x + 16$$

$$x^2 - 8x + 16$$

(collect

$$(x - 4)^2$$

$$(x - 4)^2$$

$\therefore$  centre

find grad

gradient

gradient

Equation

$\therefore$

$\therefore$



③  $x^2 + y^2 - 8x + 14y + 10 = 0$  at point  $(1, 0)$

using completing the square method

$$x^2 - 8x + y^2 + 14y + 10 = 0$$

add half the coefficient of x and y squared to both sides

$$x^2 - 8x + 16 + y^2 + 14y + 49 = -10 + 16 + 49$$

$$x^2 - 8x + 16 + y^2 + 14y + 49 = 25$$

collect the squares

$$(x - 4)^2 + (y + 7)^2 = 25$$

$$(x - 4)^2 + (y + 7)^2 = r^2$$

∴ Centre =  $(4, -7)$  and radius = 5 at point  $P = (1, 0)$

Find gradient of  $CP = \frac{y_2 - y_1}{x_2 - x_1}$  where  $y_2 = -7, y_1 = 0$   
 $x_2 = 4, x_1 = 1$

$$CP = \frac{-7 - 0}{4 - 1}$$

$$\text{gradient } CP = \frac{-7}{3}$$

$$\text{gradient of tangent} = \frac{3}{7}$$

$$\begin{aligned} \text{Equation of tangent} &= y - y_1 = m(x - x_1) \\ &= y - 0 = \frac{3}{7}(x - 1) \end{aligned}$$

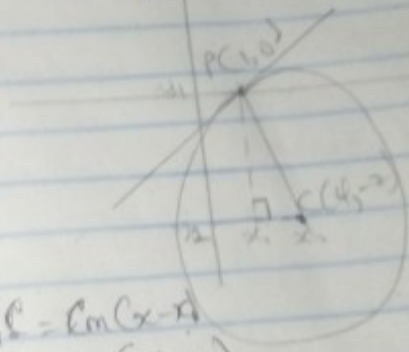
$$= y = \frac{3}{7}x - \frac{3}{7} \text{ multiply through by 7}$$

$$7y = 3x - 3$$

$$\therefore \text{The tangent's equation} = 7y = 3x - 3$$

Or

$$\text{The equation of tangent} = 7y - 3x + 3 = 0$$



squared  
+ 36

(1, 0)

by 6