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Course: Mat 102
Dept: Mechantronics
College: Engineering
Matric: 19/ENG05/064

(1) A particle moves along a curve $x = t^2$, $y = -5t^2 + t$, $z = t + 7$ where t is time. Find the acceleration

Solution

$r = x + y + z$

$r = t^2\hat{i} + (-5t^2 + t)\hat{j} + (t + 7)\hat{k}$

Velocity $= \frac{dr}{dt} = 2t\hat{i} + (-10t + 1)\hat{j} + (1)\hat{k}\ ms^{-1}$

acceleration $= \frac{dv}{dt} = \frac{d^2r}{dt^2} = 2\hat{i} - 10\hat{j}$

$\therefore$ acceleration $= 2\hat{i} - 10\hat{j}\ ms^{-2}$

(2) If $P = i - 9j - 4k$, $Q = 8i - 3j + 6k$, $R = 2i - 4j - 3k$

find $(P \times Q) \cdot (R \times P)$

Solution

$$P \times Q = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -9 & -4 \\ 8 & -3 & 6 \end{vmatrix} = \hat{i}\begin{vmatrix} -9 & -4 \\ -3 & 6 \end{vmatrix} - \hat{j}\begin{vmatrix} 1 & -4 \\ 8 & 6 \end{vmatrix} + \hat{k}\begin{vmatrix} 1 & -9 \\ 8 & -3 \end{vmatrix}$$

$= \hat{i}(-54 - 12) - \hat{j}(6 + 32) + \hat{k}(-3 + 72)$

$= -66\hat{i} - 38\hat{j} + 69\hat{k}$

$$(R \times P) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -4 & -3 \\ 1 & -9 & -4 \end{vmatrix} = \hat{i}\begin{vmatrix} -4 & -3 \\ -9 & -4 \end{vmatrix} - \hat{j}\begin{vmatrix} 1 & -3 \\ 1 & -4 \end{vmatrix} + \hat{k}\begin{vmatrix} 1 & -4 \\ 1 & -9 \end{vmatrix}$$

$= \hat{i}(+16 - 27) - \hat{j}(-4 + 3) + \hat{k}(-9 + 4)$

$= -11\hat{i} + \hat{j} - 5\hat{k}$

$\therefore (P \times Q) \cdot (R \times P) = (-66\hat{i} - 38\hat{j} + 69\hat{k}) \cdot (-11\hat{i} + \hat{j} - 5\hat{k})$

$= 726 - 38 - 345$

$= 343$

(3) Given $F = 5\cos 7t\,\hat{i} - 2e^{3t}\hat{j} - 4t^3\hat{k}$, find the integral of F with respect to t.

Solution

$$\frac{df}{dt} = \int 5\cos 7t\,\hat{i} - 2e^{3t}\,\hat{j} - 4t^3\,\hat{k}$$

$$f = \int 5\cos 7t\,\hat{i}\,dx - \int 2e^{3t}\,\hat{j} - \int 4t^3\,\hat{k}\,dx$$

$$f = \int 5\cdot\frac{1}{7}\cos 7t\,\hat{i} - \int 2\cdot\frac{1}{3}e^{3t}\,\hat{j} - \int \frac{4t^{3+1}}{3+1}\,\hat{k}$$

$$f = \frac{5}{7}\cos 7t\,\hat{i} - \frac{2}{3}e^{3t}\,\hat{j} - \frac{\cancel{4}t^4}{\cancel{4}}\,\hat{k}$$

$$\therefore f = \frac{5}{7}\cos 7t\,\hat{i} - \frac{2}{3}e^{3t}\,\hat{j} - t^4\,\hat{k} + c$$