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Day Thursday
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Course Mat 102
Dept Mechantronics
College Engineering
Matric 19/ENG05/064

(1.) Find the equation of the tangent at the point (1,0) on the circle $x^2+y^2-5x-y+6=0$

(2.) Find the equation of the tangent at the point (4,0) on the circle $x^2+y^2-12x-12y+47=0$

(3.) Find the equation of the tangent at the point (5,0) on the circle $x^2+y^2-8x+16y+40=0$.

Solution

(1.) $x^2+y^2-5x-y+6=0$

using completing the square method

$x^2-5x+y^2-y=-4$

add half the coefficient of x and y to both sides squared

$x^2-5x+\frac{25}{4}+y^2-y+\frac{1}{4}=-4+\frac{25}{4}+\frac{1}{4}$

collect the squares

$\left(x-\frac{5}{2}\right)^2+\left(y-\frac{1}{2}\right)^2=\frac{-16+25+1}{4}=\frac{10}{4}$

Formula for centre and radius of circle

$=(x-h)+(x-k)=r^2$

where (h,k) = centre and r = radius

∴ The centre of circle = $(5/2, 1/2)$ at point $(1,0)$

and radius = $\sqrt{10/4}=\frac{\sqrt{10}}{2}$ "

Find the gradient of P

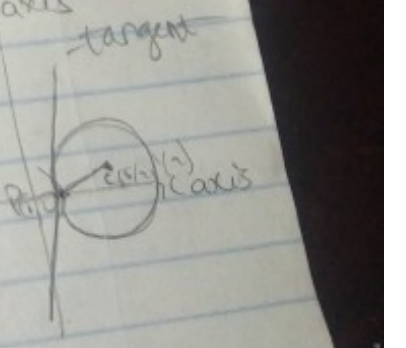
$m=\frac{(y_2-y_1)}{(x_2-x_1)}$ where $y_2=1/2, y_1=0$

$x_2=5/2\ x_1=1$

$m=\frac{1/2}{\frac{5}{2}-1}=\frac{1/2}{\frac{5-2}{2}}=\frac{1}{2}\div\frac{3}{2}$

$m=\frac{1}{2}\times\frac{2}{3}$

$m=1/6=\frac{1}{6}$



gradient of tangent = -3 //

Equation of tangent = $y - y_1 = m(x - x_1)$

$= y - 0 = -3(x - 1)$

$= y = -3x + 3$ //

(2) $x^2 + y^2 - 12x - 12y + 47 = 0$

Using completing the square method.

$x^2 - 12x + y^2 - 12y = -47$

add half the coefficient of x and y squared to both side.

$x^2 - 12x + 36 + y^2 - 12y + 36 = -47 + 36 + 36$

collect the Squares

$(x - 6)^2 + (y - 6)^2 = 25$

Using the centre and radius formula which is

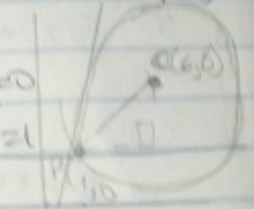
$(x - h)^2 + (y - k)^2 = r^2$

where centre = (h, k) and radius = r

∴ centre = (6, 6) and radius = 5 // at P = (1, 0)

find the gradient of CP

gradient CP = $\frac{y_2 - y_1}{x_2 - x_1}$ $y_2 = 6; y_1 = 0$ $x_2 = 6; x_1 = 1$



gradient CP = $\frac{6 - 0}{6 - 1} = \frac{6}{5}$

∴ the gradient of tangent

$m = -\frac{5}{6}$ //

The equation $y - y_1 = m(x - x_1)$

$y - 0 = m(x - 1)$

$y = m(x - 1)$

$y = -\frac{5}{6}(x - 1)$

$y = -\frac{5}{6}x + \frac{5}{6}$ multiply by 6

∴ The equation of tangent of circle $6y = -5x + 5$ // or // $6y + 5x - 5 = 0$ //

(a) $x^2 + y^2 - 8x + 14y + 40 = 0$ at point $(1, 0)$

using completing the square method

$x^2 - 8x + y^2 + 14y = -40$

add half the coefficient of x and y squared to both sides

$x^2 - 8x + 16 + y^2 + 14y + 49 = -40 + 16 + 49$

$x^2 - 8x + 16 + y^2 + 14y + 49 = 25$

collect the squares

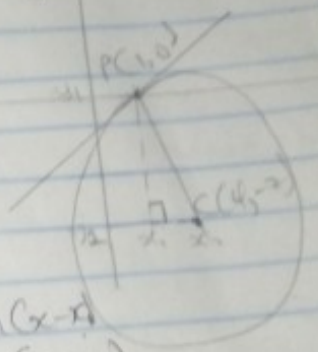
$(x - 4)^2 + (y + 7)^2 = 25$ u

$(x - h)^2 + (y - k)^2 = r^2$

$\therefore$ Centre $= (4, -7)$ and radius $= 5$ at point $P = (1, 0)$

Find gradient of P, $P = \frac{y_2 - y_1}{x_2 - x_1}$ where $y_2 = -7$ & $y_1 = 0$ & $x_2 = 4$ & $x_1 = 1$

$\therefore P = \frac{-7 - 0}{4 - 1}$



gradient $P = -\frac{7}{3}$

gradient of tangent $= \frac{3}{7}$ u

Equation of tangent $= y - y_1 P = m(x - x_1)$

$= y - 0 P = \frac{3}{7}(x - 1)$

$= y = \frac{3}{7}x - \frac{3}{7}$ multiply through by 7

$7y = 3x - 3$

$\therefore$ The tangent's equation $= 7y = 3x - 3$

Or

The equation of tangent $= 7y - 3x + 3 = 0$ u