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COMPUTER ENGINEERING (19/ENG02/054)
MAT 104 Assignment.
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1) Integrate $3te^{2t}$ with respect to t

Solution

$\int dy = \int 3te^{2t}\,dt$

$\int dy = 3\int te^{2t}\,dt$ (Since 3 is a constant)

Solving for $\int te^{2t}\,dt$ (we solve by part)

$\therefore \int t = 1$

$\int e^{2t} = \frac{e^{2t}}{2}$

$$= \frac{te^{2t}}{2} - \int \frac{e^{2t}}{2}\,dt \quad \text{—— eqn I}$$

Solving for $\int e^{2t}/2\,dt = \frac{1}{2}\int e^{2t}\,dt$

let $u = 2t \quad \therefore \quad du/dt = 2$

$dt = du/2$

$\frac{1}{2}\int e^{u}\,dt$ (but $dt = du/2$)

$\therefore \frac{1}{4}\int e^{u}\,du$

$= \frac{1}{4} \cdot e^{u}$ recall that $u = 2t$

$= \frac{e^{2t}}{4}$

from eqn I $= \frac{te^{2t}}{2} - \int \frac{e^{2t}}{2}\,dt$

we have that $= \frac{te^{2t}}{2} - \frac{e^{2t}}{4}$

$$y = 3\left[\frac{te^{2t}}{2} - \frac{e^{2t}}{4}\right] + c$$

$$y = 3\left[\frac{2te^{2t} - e^{2t}}{4}\right] + c$$

$$y = \frac{3(2t-1)e^{2t}}{4} + C$$

2 $x^2 \sin x$ ⇒ Integrale with respect to x

Solution

Integrate by parts $\int uv' = uv - \int u'v$ ∴ $\int u\,dv = uv - \int v\,du$

where $u = -x^2$ $\quad v' = \sin x$

$du/dx = 2x$ $\quad dv/dx = -\cos x$

$= -x^2 \cos x - \int -2x\cos(x)\,dx$ — (I)

Solving for $\int -2x\cos x\,dx$

$-2\int x\cos x\,dx$

Integrate by part

$= -2[x\sin x - (-\cos x)]$

$= -2[x\sin x + \cos x]$

$= -2x\sin x - 2\cos x$

Put the answer back in eqn I

$-x^2\cos x - [-2x\sin x - 2\cos x]$

$-x^2\cos x + 2x\sin x + 2\cos x$

∴ $\int x^2 \sin x = 2x\sin x - x^2\cos x + 2\cos x + C$

Simplified as ⇒ $2x\sin x + (2 - x^2)\cos x + C$

3) $\int \sin 7x \cos 2x$ ⇒ integrate with respect to x

Solution

let $7x = A$ and $2x = B$

recall that $\int \sin A \cos B = \frac{1}{2}\int[\sin(A+B) - \sin(A-B)]$

$= \frac{1}{2}\int[\sin(7x+2x) - \sin(7x-2x)]$

$= \frac{1}{2}\int[\sin 9x - \sin 5x]$

$= \frac{1}{2}\int \sin(9x)\,dx + \frac{1}{2}\int \sin(5x)\,dx$

Solving $\frac{1}{2}\int \sin 9x\,dx = \frac{1}{2}\left[-\frac{\cos 9x}{9}\right] = -\frac{\cos 9x}{18}$

$\frac{1}{2}\int \sin 5x\,dx = \frac{1}{2}\left[-\frac{\cos 5x}{5}\right] = -\frac{\cos 5x}{10}$

Therefore $\int \cos 2x \sin 7x\,dx = -\frac{\cos(9x)}{18} - \frac{\cos(5x)}{10} + C$

4 Integrate $\frac{2x-3x^2}{(1-x)}$ with respect to x

Solution

Simplify $\int \frac{2x-3x^2}{(1-x)}$

$$\int \frac{x(3x-2)}{x-1}\,dx$$

let $u = x-1$; $du/dx = 1$

$\therefore du = dx$

$$= \int \frac{\cancel{x}(3x-2)}{u}\,du$$

but if $u = x-1$, then $x = u+1$

$$\therefore = \int \frac{(u+1)(3(u+1)-2)}{u}\,du$$

$$\int \frac{(u+1)(3u+1)}{u}\,du \Rightarrow \int \frac{3u^2+4u+1}{u}\,du$$

$$\int \left[3u + 4 + \frac{1}{u}\right] du$$

$$= \int 3u\,du + \int 4\,du + \int 1/u\,du$$

$$= 3\int u\,du + 4\int 1\,du + \int 1/u\,du$$

$$= \left(\frac{3.u^2}{2}\right) + 4u + \ln u$$

$$= \frac{3u^2}{2} + 4u + \ln u$$

(Recall that $u = x-1$)

$$= \frac{3(x-1)^2}{2} + 4(x-1) + \ln(x-1)$$