

$3te^{2t}$

Let $u = 3t$ & $dv = e^{2t}$
 $\frac{du}{dt} = 3$ and $\int dv = \frac{1}{2} e^{2t}$
 $du = 3dt$ $v = \frac{e^{2t}}{2}$

Using $uv - \int v du = \int u dv$
 $= 3t \left(\frac{e^{2t}}{2} \right) - \int \frac{e^{2t}}{2} \times 3 dt$

$3t \left(\frac{e^{2t}}{2} \right) - \frac{3}{2} \int e^{2t} dt$

$3t \left(\frac{e^{2t}}{2} \right) - \frac{3}{2} \times \frac{e^{2t}}{2} + C$

$\left[\frac{3te^{2t}}{2} - \frac{3e^{2t}}{4} \right] + C //$

$x^2 \sin x$

Let $u = x^2$ & $dv = \sin x$
 $\frac{du}{dx} = 2x$ and $v = -\cos x$

Using $uv - \int v du$
 $(x^2)(-\cos x) - \int (-\cos x)(2x dx)$
 $-x^2 \cos x - \int -2x \cos x dx$

Let $u = 2x$ & $dv = \cos x$
 $\frac{du}{dx} = 2$ and $v = \sin x$

$\therefore (-2x)(\sin x) - \int (\sin x)(2) dx$
 $-2x \sin x - 2 \int \sin x dx$
 $-2x \sin x - (-2) = -2x \sin x + 2 + C$

$-2x \sin x - 2 \cos x + C$

$\therefore \int x^2 \sin x = -x^2 \cos x - 2x \sin x - 2 \cos x + C //$

$\int \sin 7x \cos 2x$

Let $A = 7x$ and $B = 2x$

$\therefore \sin A \cos B = \frac{1}{2} (\sin(A+B) + \sin(A-B))$

$\frac{1}{2} (\sin(7x+2x) + \sin(7x-2x))$

$\frac{1}{2} (\sin 9x + \sin 5x)$

$\int \sin 7x \cos 2x dx = \frac{1}{2} \int (\sin 9x + \sin 5x)$
 $= \frac{1}{2} \left(-\frac{\cos 9x}{9} - \frac{\cos 5x}{5} \right)$

$\frac{\cos 9x}{18} + \frac{\cos 5x}{10} + C //$

$\frac{2x-3x^2}{1-x}$

$\frac{2x-3x^2}{1-x} = \frac{2x-2x^2 - x^2}{1-x}$
 $= \frac{-2x-2x^2}{1-x} = \frac{-x^2}{1-x} = \frac{-x^2 + x^3}{-x^3}$

which can now be

$\int \frac{(2x-x^2)}{1-x} dx + \int \frac{x^3}{1-x} dx$
 $= \frac{2x^2}{2} - \frac{x^3}{3} + x^3 \ln(1-x) //$