ILODIBE ANTHONY UDENNA
COMPUTER ENGINEERING
19/ENG02/026
MAT 104 SERIAL NO: 35

Integrate the following with respect to their variable

1. $3te^{2t}$
2. $x^2 \sin x$
3. $\sin 7x \cos 2x$
4. $\frac{(2x-3x^2)}{1-x}$

Sol

1. $\int dy = \int 3te^{2t}\,dt$

$\int dy = 3\int te^{2t}\,dt$ (Since 3 is a constant)

Solving for $\int te^{2t}\,dt$ (we solve by part)

$\therefore \int t = 1$

$\int e^{2t} = \frac{e^{2t}}{2}$

$= \frac{te^{2t}}{2} - \int \frac{e^{2t}}{2}\,dt$ ——— equ ①

Solving for $\int \frac{e^{2t}}{2}\,dt = \frac{1}{2}\int e^{2t}\,dt$

let $u = 2t$ ; $\frac{du}{dt} = 2$

$\therefore dt = \frac{du}{2}$

$\frac{1}{2}\int e^u\,dt$ (but $dt = du/2$)

$= \frac{1}{2}\int e^u \cdot du/2$

$= \frac{1}{4}\int e^u \cdot du$

$= \frac{1}{4} \cdot e^u$

recall that $u = 2t$

$= \frac{e^{2t}}{4}$

from equ ① $= \frac{te^{2t}}{2} - \int \frac{e^{2t}}{2} dt$

we have that $= \frac{te^{2t}}{2} - \frac{e^{2t}}{4}$

$y = 3\left[\frac{te^{2t}}{2} - \frac{e^{2t}}{4}\right] + c$

$y = \left[\frac{3te^{2t}}{2} - \frac{3e^{2t}}{4}\right] + c$

2 $\int x^2 \sin x \Rightarrow$ Integrate with respect to $x$

Sol

Integrate by parts $\int uv' = uv - \int u'v$ $\therefore \int u dv = uv - \int v du$

where $u = x^2$ $v' = \sin x$

$\frac{du}{dx} = 2x$ $\frac{dv}{dx} = -\cos x$

$= -x^2 \cos x - \int -2x \cos(x) dx$ —— ①

Solving for $\int -2x \cos x dx$

$-2 \int x \cos x dx$

Integrate by parts

$= -2[x \sin x - (-\cos x)]$

$= -2[x \sin x + \cos x]$

$= -2x \sin x - 2\cos x$

Put the answer back in equ ①

$-x^2 \cos x - [-2x \sin x - 2\cos x]$

$-x^2 \cos x + 2x \sin x + 2\cos x$

$\int x^2 \sin x = 2x \sin x - x^2 \cos x + 2\cos x + C$

3 $\int \sin 7x \cos 2x \Rightarrow$ Integrate with respect to $x$

Sol

let $7x = A$ and $2x = B$

recall that $\int \sin A \cos B = \frac{1}{2}\int\left[\sin(A+B) - \sin(A-B)\right]$

$= \frac{1}{2}\int\left[\sin(7x + 2x) - \sin(7x - 2x)\right]$

$$= \frac{1}{2}\int [\sin 9x - \sin 5x]$$

$$= \tfrac{1}{2}\int \sin 9x \, dx + \tfrac{1}{2}\int \sin 5x \, dx$$

Solving $\tfrac{1}{2}\int \sin 9x \, dx = \tfrac{1}{2}\left[-\frac{\cos 9x}{9}\right] = -\cos 9x/18$

Solving $\tfrac{1}{2}\int \sin 5x \, dx = \tfrac{1}{2}\left[-\frac{\cos 5x}{5}\right] = -\cos 5x/10$

$$\therefore \int \cos 2x \sin 7x \, dx = \frac{-\cos 9x}{18} - \frac{\cos 5x}{10} + C$$

4 $$\int \frac{2x - 3x^2}{1-x}$$

$$\begin{array}{r|l} & 2x - x^2 \\ \hline 1-x & 2x - 3x^2 \\ & -(2x - 2x^2) \\ \hline & -x^2 \\ & -(-x^2 + x^3) \\ \hline & x^3 \end{array}$$

which can be

$$\int (2x - 3x^2)\, dx + \int \frac{x^3}{(1-x)} \, dx$$

$$= \frac{\cancel{2}x^2}{\cancel{2}} - \frac{\cancel{3}x^3}{\cancel{3}} + x^3 \ln(1-x)$$

$$= x^2 - x^3 + x^3 \ln(1-x)$$