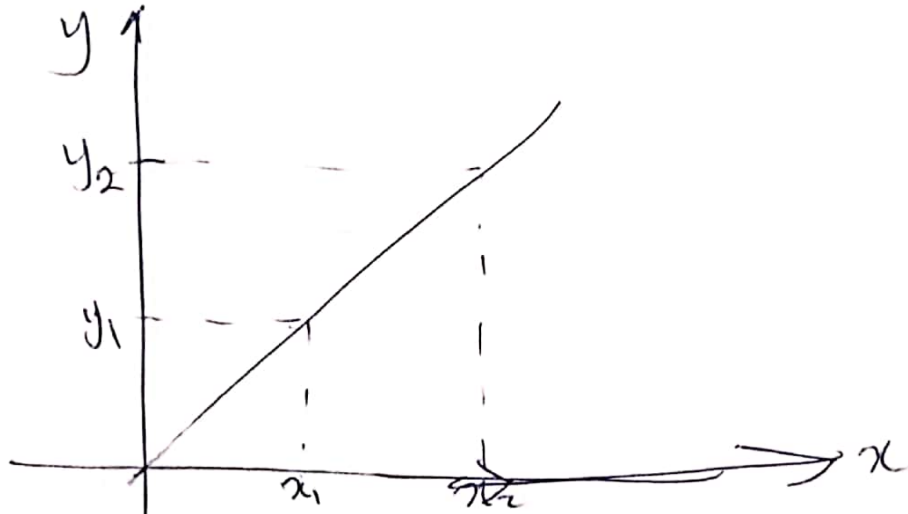
# Equations of a Straight Line



## 1. Through the Origin




The gradient

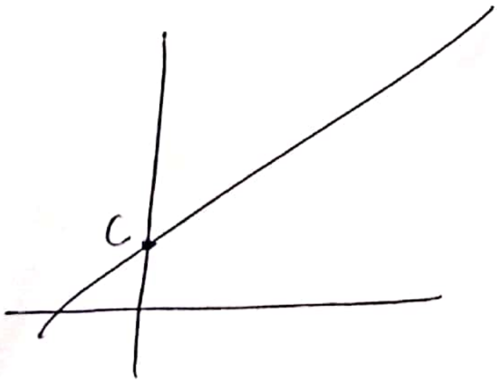
$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$\therefore$ Equation of a line passing through origin with gradient m is

$$y = mx$$

Eg $y = 2x$

## 2. With intercept on the y axis




$$y = mx + c$$

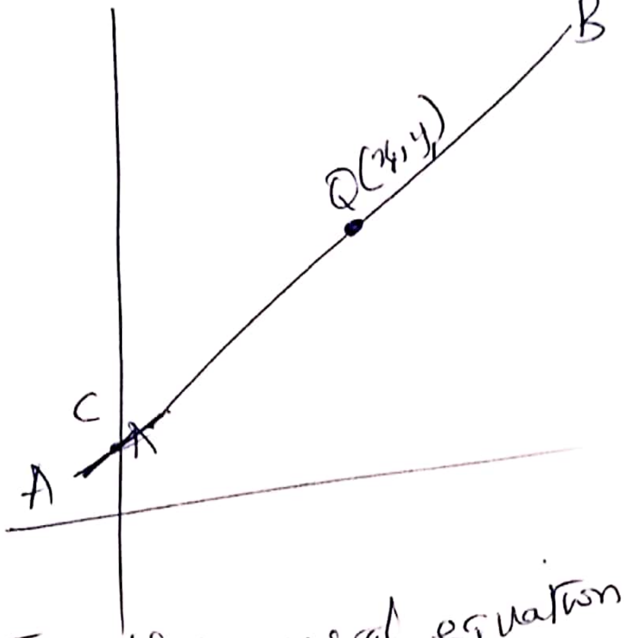
Eg (1) $y = 3x + 2$
gradient – 3
Intercept – 2

(2) $y = -2x + 4$

(3) $2y - 10x = 8$

4) $x + y + 1 = 0$

(3) Equation of a line passing through a given point




From the general equation

$$y = mx + c \quad \text{———} *$$

but the line passes through $Q(x_1, y_1)$, then

$$y_1 = mx_1 + c \quad \text{——} *$$

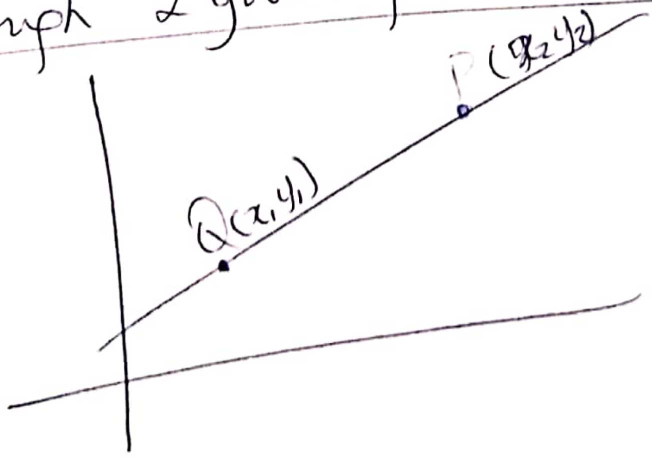
$$\Rightarrow \quad c = y_1 - mx_1$$

Hence $*$ becomes

$$y = mx + y_1 - mx_1$$

$$y - y_1 = m(x - x_1)$$

(4) Through 2 given points



In order to find the equation of a straight line passing through points $(x_1, y_1)$ and $(x_2 y_2)$ first We don't know the gradient, so we find the gradient with these 2 points

$$m = \frac{y_2 - y_1}{x_2 - x_1} \quad —— \quad *$$

Now we have m, we can use one of the points to find the equation, taking point $(x_1 y_1)$ i.e. from the previous subtopic

$$y - y_1 = m(x - x_1) \quad — \quad ii$$

Substituting * into 2*

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$\Rightarrow \quad \frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1} \quad — \quad (**)$$

# The General Equation of a Straight Line

$$ax + by + c = 0$$

We have written the equation of a straight line in this form for some examples above. Also there are some special cases

→ If $a = 0$, then $by + c = 0$ — no term in $x$

$$\Rightarrow y = -\frac{c}{b}$$

hence line is horizontal


→ If $b = 0$ then,

$$ax + c = 0$$

$$\Rightarrow x = -\frac{c}{a}$$

then the line is vertical and obviously the equation cannot be written in the form $y = mx + c$

We see that the equation

$ax + by + c = 0$ is the most general equation for a straight line and can be used where other forms of equations of straight lines fail.