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Question 1

Find the equation of the tangent at the point $(1, 0)$ on the circle $x^2 + y^2 - 5x - y + 4 = 0$

Solution

$$x^2 + y^2 - 5x - y + 4 = 0$$

$$x^2 - 5x + y^2 - y + 4 = 0$$

$$(x - 2.5)^2 - 6.25 + (y - 0.5)^2 - 0.25 + 4 = 0$$

$$(x - 2.5)^2 + (y - 0.5)^2 = 2.4$$

Note: $(x - x_1)^2 + (y - y_1)^2 = r^2$

∴ Centre of circle $(2.5, 0.5)$; radius $\sqrt{2.4}$

$$\text{Gradient of radius} = \frac{0 - 0.5}{1 - 2.5} = \frac{-0.5}{-1.5} = 0.3 = \frac{1}{3}$$

∴ Gradient of tangent $= -3$

∴ Equation of the tangent at point $(1, 0)$ is

$$y - 0 = -3(x - 1)$$

$$y = -3x + 3$$

$$\boxed{y + 3x = 3}$$

Question 2

Find the equation of the tangent at the point $(1, 0)$ on the circle $x^2 + y^2 - 12x + 12y + 47 = 0$

Solution

$$x^2 + y^2 - 12x - 12y + 47 = 0$$

$$x^2 - 12x + y^2 - 12y + 47 = 0$$

$$(x-6)^2 - 36 + (y-6)^2 - 36 + 47 = 0$$

$$(x-6)^2 + (y-6)^2 = 25$$

Centre of circle $(6, 6)$; radius $= \sqrt{25} = 5$

$$\text{Gradient of radius} = \frac{0-6}{1-6} = \frac{-6}{-5} = \frac{6}{5}$$

$$\text{Gradient of tangent} = \frac{-5}{6}$$

$$\text{Equation of tangent} = y - 0 = \frac{-5}{6}(x - 1)$$

$$y = -\frac{5}{6}x + \frac{5}{6}$$

$$y = -0.83x + 0.83$$

$$\boxed{y + 0.83x = 0.83}$$

Question 3

Find the equation of the tangent at the point $(1, 0)$ on the circle $x^2 + y^2 - 8x + 14y + 40 = 0$

Solution

$$x^2 + y^2 - 8x + 14y + 40 = 0$$

$$x^2 - 8x + y^2 + 14y + 40 = 0$$

$$(x - 4)^2 - 16 + (y + 7)^2 - 49 + 40 = 0$$

$$(x - 4)^2 + (y + 7)^2 = 25$$

Centre of circle $(4, -7)$ radius $= \sqrt{25} = 5$

$$\text{Gradient of radius} = \frac{0 - (-7)}{1 - 4} = \frac{7}{-3}$$

$$\therefore \text{Gradient of tangent} = \frac{3}{7}$$

$$\text{Equation of tangent} \Rightarrow y - 0 = \frac{3}{7}(x - 1)$$

$$\boxed{y = \frac{3}{7}x - \frac{3}{7}}$$