

1. $3te^{2t}$

let $u = 3t$

$du = e^{2t}$

$\frac{du}{dt} \times 3$

$\int du = \int e^{2t}$

$v = \frac{e^{2t}}{2}$

$du = 3dt$

Using $uv - \int v du = \int u dv$

$= 3t \left(\frac{e^{2t}}{2} \right) - \int \frac{e^{2t}}{2} \times 3dt$

$3t \left(\frac{e^{2t}}{2} \right) - \frac{1}{2} \int 3e^{2t} dt$

$3t \left(\frac{e^{2t}}{2} \right) - \frac{1}{2} \times \frac{3e^{2t}}{2} + C$

$\left[\frac{3te^{2t}}{2} - \frac{3e^{2t}}{4} \right] + C$

2. $\int x^2 \sin x$

let $u = x^2$ and $dv = \sin x$

$\frac{du}{dx} = 2x$ and $v = -\cos x$

using $uv - \int v du$

$(x^2)(-\cos x) - \int (-\cos x)(2x dx)$

$-x^2 \cos x - \int -2x \cos x dx$

let $u = -2x$ and $du = -2 dx$

$\frac{du}{dx} = -2$ and $v = \sin x$

$\therefore (-2x)(\sin x) - \int (\sin x)(-2) dx$

$-2x \sin x - (-2) \int \sin x dx$

$-2x \sin x - (-2)(-\cos x) + C$

$-2x \sin x - 2 \cos x + C$

$\therefore \int x^2 \sin x = -x^2 \cos x -$

$2x \sin x - 2 \cos x + C$

$$3 \int \sin 7x \cos 2x$$

$$\text{let } A = 7x, \quad B = 2x$$

$$\int \sin 7x \cos 2x = \frac{1}{2} \left[\sin(7x + 2x) + \sin(7x - 2x) \right]$$

$$\int \sin 7x \cos 2x = \frac{1}{2} \left[\sin 9x + \sin 5x \right]$$

$$= \frac{1}{2} \int \left[\frac{\sin 9x}{9} + \frac{\sin 5x}{5} \right]$$

$$= \frac{-\cos 9x}{18} - \frac{\cos 5x}{10} + C$$

$$4 \frac{2x - 3x^2}{1-x}$$

$$\begin{array}{r} 2x - 3x^2 \\ 1-x \overline{) 2x - 3x^2} \\ \underline{-2x - 2x^2} \\ -x^2 \\ \underline{-x^2 + x^3} \\ -x^3 \end{array}$$

Which can now be

$$\int (2x - 3x^2) dx + \int \frac{x^3}{1-x} dx$$

$$= \frac{2x^2}{2} - \frac{3x^3}{3} + x^3 \ln(1-x)$$