

1.) $3te^{2t} dt$

$= ut \quad v = 3t, \quad dv = 3e^{2t}$

$\frac{dv}{dt} = 3, \quad v = \int e^{2t} dt = u + v = 2t, \quad \frac{dv}{dt} = 2 \quad dt = \frac{dv}{2}$
 $= \int e^u \frac{du}{2} = \frac{1}{2} \int e^u du = \frac{1}{2} e^u = \frac{e^{2t}}{2}$

$\frac{dv}{dt} = 3, \quad v = \frac{e^{2t}}{2} = \int u dv = uv - \int v du$

$\int 3te^{2t} dt = 3t \left(\frac{e^{2t}}{2} \right) - \frac{1}{2} \int e^{2t} (3)$

$= \frac{3}{2} te^{2t} - \frac{3}{2} \int e^{2t} dt$

$= \frac{3}{2} te^{2t} - \frac{3}{2} \left(\frac{e^{2t}}{2} \right) + C$

$= \frac{3}{2} te^{2t} - \frac{3e^{2t}}{4} + C = \frac{3}{2} e^{2t} \left(t - \frac{1}{2} \right) + C$

2.) $\int x^2 \sin x dx$

let $u = x^2, \quad dv = \sin x$

$\frac{du}{dx} = 2x \quad v = \int \sin x dx = -\cos x$

$\int x^2 \sin x dx = (x^2)(-\cos x) - \int (-\cos x)(2x)$

$= -x^2 \cos x + 2 \int x \cos x dx$

$= -x^2 \cos x + 2(x \sin x - (-\cos x)) + C$

$= x \sin x + \cos x + C$

$= -x^2 \cos x + 2(x \sin x + \cos x) + C$

$= -x^2 \cos x + 2x \sin x + 2 \cos x + C$

$= (2-x^2) \cos x + 2x \sin x + C$

3.) $\int \sin 7x \cos 2x$

let $u = 7x, \quad v = 2x$

$$\sin A \cos B = \frac{1}{2} [\sin(A+B) + \sin(A-B)]$$

$$= \frac{1}{2} [\sin(7x+2x) + \sin(7x-2x)]$$

$$= \frac{1}{2} (\sin 9x + \sin 5x)$$

$$= \frac{1}{2} \left[\int \sin 9x \, dx + \int \sin 5x \, dx \right]$$

$$= \frac{1}{2} \left[-\frac{\cos 9x}{9} - \frac{\cos 5x}{5} \right]$$

$$= -\frac{\cos 9x}{18} - \frac{\cos 5x}{10} + C$$

$$4.) \int \frac{2x - 3x^2 \, dx}{1-x}$$

$$= \int \frac{-3x^2 + 2x \, dx}{-x+1}$$

$$= \int \frac{-(3x^2 - 2x)}{-(-x-1)}$$

$$= \int \frac{3x^2 - 2x}{x-1}$$

$$\begin{array}{r} 3x+1 \\ x-1 \overline{) 3x^2 - 2x} \\ \underline{-3x^2 - 3x} \\ x \end{array}$$

$$\underline{-x-1}$$

$$1$$

$$= \int (3x+1) \, dx + \int \frac{1}{x-1} \, dx$$

$$= \frac{3x^2 + x}{2} + \ln|x-1| + C$$