

1) $\int 3te^{2t}$

$u = 3t \quad dv = e^{2t}$
 $du = 3 \quad v = \frac{e^{2t}}{2}$
 $\Rightarrow \int u dv = uv - \int v du$
 $\therefore \int u dv = 3t \frac{e^{2t}}{2} - \int \frac{3e^{2t}}{2}$
 $\Rightarrow \frac{3te^{2t}}{2} - \frac{3e^{2t}}{4} + C$

3) $\sin 7x \cos 2x$

Remember $A = 7x \quad B = 2x$
 $\sin A \cos B = \frac{1}{2} [\sin(A+B) + \sin(A-B)]$
 $\Rightarrow \frac{1}{2} [\sin(7x+2x) + \sin(7x-2x)]$
 $\Rightarrow \frac{1}{2} (\sin 9x + \sin 5x)$
 $\Rightarrow \frac{1}{2} \left(\frac{\sin 9x}{9} + \frac{\sin 5x}{5} \right)$
 $\Rightarrow \frac{\sin 9x}{18} + \frac{\sin 5x}{10}$

2) $\int x^2 \sin x$

$u = x^2 \quad dv = \sin x$
 $du = 2x \quad v = -\cos x$
 $\therefore \int u dv = uv - \int v du$
 $\Rightarrow -x^2 \cos x + \int 2x \cos x$

4) $\frac{(2x-3x^2)}{1-x}$

~~$\frac{3x^2+2x}{1-x}$~~
 $\Rightarrow \frac{-x+1}{1-x} - \frac{3x^2+2x}{1-x}$
 $\frac{3x+1}{1-x} \begin{array}{r} -3x^2+2x+0 \\ -3x^2+3x+0 \\ \hline -x+1 \\ \hline -x+1 \\ \hline -10 \end{array}$
 $\Rightarrow \frac{3x+1}{1-x} + \frac{-1}{2x+3x^2}$

$dv = \cos x \quad \text{let } u = 2x$
 $v = \sin x \quad du = 2$
 $\Rightarrow 2x \sin x - \int 2 \sin x$
 $\int u dv = x^2 \cos x + 2x \sin x - \int 2 \sin x$
 $\therefore \Rightarrow -x^2 \cos x + 2x \sin x - (-2 \cos x)$
 $+ C$
 $\Rightarrow -x^2 \cos x + 2x \sin x + 2 \cos x + C$

$$v = 2x$$

$$\frac{dv}{dx} = 2 \quad \frac{dv}{v} = \frac{2}{v}$$

$$\int \frac{2}{v} dx = \ln v$$

$$\Rightarrow \frac{3x^2}{2} + x + \left[\frac{-1}{2x+3x^2} \right]$$

$$\frac{-1}{2x+3x^2} \Rightarrow \frac{A}{x} + \frac{B}{2+3x}$$

$$\therefore -1 = A(2+3x) + B(x)$$

When $x=0$

When $x = -\frac{2}{3}$

$$-1 = 2A$$

$$\therefore A = -\frac{1}{2}$$

$$\Rightarrow -1 = A \left(2 + 3 \left(-\frac{2}{3} \right) \right) + B \left(-\frac{2}{3} \right)$$

$$\Rightarrow \therefore \frac{-2B}{3} = -1$$

$$\therefore B = \frac{3}{2}$$

$$\Rightarrow \frac{3x^2}{2} + x + \int \frac{-1}{2x} + \int \frac{3}{4+6x}$$

$$\Rightarrow \frac{3x^2}{2} + x - \frac{1}{2} (\ln)x + \frac{1}{2} \ln(4+6x) + C$$