

$$1.) \int 3te^{2t}$$

Solution

$$\text{let } u = 3t$$

$$\text{and } dv = e^{2t}$$

$$\frac{du}{dt} = 3$$

$$\int dv = \int e^{2t}$$

$$du = 3dt$$

$$v = \frac{e^{2t}}{2}$$

$$\text{Using } uv - \int v du = \int u dv$$

$$= 3t \left(\frac{e^{2t}}{2} \right) - \int \frac{e^{2t}}{2} \times 3 dt$$

$$3t \left(\frac{e^{2t}}{2} \right) - \frac{1}{2} \int 3e^{2t} dt$$

$$3t \left(\frac{e^{2t}}{2} \right) - \frac{1}{2} \times \frac{3e^{2t}}{2} + C$$

$$\left[\frac{3te^{2t}}{2} - \frac{3e^{2t}}{4} \right] + C$$

$$2.) \int x^2 \sin x$$

$$\text{let } u = x^2 \text{ and } dv = \sin x$$

$$\frac{du}{dx} = 2x \text{ and } v = -\cos x$$

$$\text{using } uv - \int v du$$

$$(x^2)(-\cos x) - \int (-\cos x)(2x dx)$$

$$-x^2 \cos x - \int -2x \cos x dx$$

$$\left[\text{let } u = -2x \text{ and } dv = \cos x \right]$$

$$\left[\frac{du}{dx} = -2 \text{ and } v = \sin x \right]$$

$$\therefore (-2x)(\sin x) - \int (\sin x)(-2) dx$$

$$-2x \sin x - (-2) \int \sin x dx$$

$$-2x \sin x - (-2) \left(-\cos x \right) + C$$

$$-2x \sin x - 2 \cos x + C$$

$$\therefore \int x^2 \sin x = -x^2 \cos x - 2x \sin x - 2 \cos x + C$$

$$3) \int \sin 7x \cos 2x$$

$$\text{let } A = 7x \text{ and } B = 2x$$

$$\therefore \text{using } = \frac{1}{2} (\sin(A+B) + \sin(A-B))$$

$$\frac{1}{2} (\sin(7x+2x) + \sin(7x-2x))$$

$$\frac{1}{2} (\sin 9x + \sin 5x)$$

$$\int \sin 7x \cos 2x dx = -\frac{1}{2} \int (\sin 9x + \sin 5x)$$
$$= -\frac{1}{2} \left[\frac{-\cos 9x}{9} - \frac{\cos 5x}{5} \right]$$

$$\frac{\cos 9x}{18} + \frac{\cos 5x}{10} + C$$

$$3) \int \sin 7x \cos 2x$$

$$\text{let } A = 7x, \quad B = 2x$$

$$\int \sin 7x \cos 2x = \frac{1}{2} [\sin(7x+2x) + \sin(7x-2x)]$$

$$\int \sin 7x \cos 2x = \frac{1}{2} [\sin 9x + \sin 5x]$$

$$= \frac{1}{2} \left[\frac{\sin 9x}{9} + \frac{\sin 5x}{5} \right]$$

$$= \left(\frac{\sin 9x}{18} + \frac{\sin 5x}{10} \right) + C$$

$$= -\frac{\cos 9x}{18} - \frac{\cos 5x}{10} + C$$