SN 97

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Math 104
Mechanical Engineering.

(1) $\int 3te^{2t}\,dt$

$= $ Let $u = 3t,\ dv = e^{2t}$

$\frac{du}{dt} = 3$, $v = \int e^{2t}\,dt =$ Let $u = 2t,\ \frac{du}{dt} = 2,\ dt = \frac{du}{2}$

$= \int e^{u}\frac{du}{2} = \frac{1}{2}\int e^{u}\,du$

$= \frac{1}{2}e^{u} = \frac{e^{2t}}{2}$

$\frac{du}{dt} = 3$, $v = \frac{e^{2t}}{2}$

$\int u\,dv = uv - \int v\,du$

$\int 3te^{2t}\,dt = 3t\left(\frac{e^{2t}}{2}\right) - \frac{1}{2}\int e^{2t}(3)$

$= \frac{3te^{2t}}{2} - \frac{3}{2}\int e^{2t}\,dt$

$= \frac{3}{2}te^{2t} - \frac{3}{2}\left[\frac{e^{2t}}{2}\right] + C$

$= \frac{3}{2}te^{2t} - \frac{3e^{2t}}{4} + C = \frac{3}{2}e^{2t}\left(t - \frac{1}{2}\right) + C$

(2) $\int x^2 \sin x\,dx$

Let $u = x^2,\ dv = \sin x$

$\frac{du}{dx} = 2x$ $v = \int \sin x\,dx = -\cos x$

$\int x^2 \sin x\,dx = (x^2)(-\cos x) - \int(-\cos x)(2x)$

$= -x^2\cos x + 2\left[\int x\cos x\,dx\right]$

$= -x^2\cos x + 2$ $\big[x\sin x - (-\cos x) + c$

$= x\sin x + \cos x + c$

$= -x^2\cos x + 2[x\sin x + \cos x] + c$

$= -x^2\cos x + 2x\sin x + 2\cos x + C$

$= (2 - x^2)\cos x + 2x\sin x + C$

(3) $\int \sin 7x \cos 2x$

Let $A = 7x$, $B = 2x$

$\sin A \cos B = \frac{1}{2}\left[\sin(A+B) + \sin(A-B)\right]$

$= \frac{1}{2}\left[\sin(7x+2x) + \sin(7x-2x)\right]$

$= \frac{1}{2}\left[\sin 9x + \sin 5x\right]$

$= \frac{1}{2}\left[\int \sin 9x \, dx + \int \sin 5x \, dx\right]$

$= \frac{1}{2}\left[-\frac{\cos 9x}{9} - \frac{\cos 5x}{5}\right]$

$= -\frac{\cos 9x}{18} - \frac{\cos 5x}{10} + C$

(4) $\int \frac{2x - 3x^2}{1-x} \, dx$

$= \int \frac{-3x^2 + 2x}{-x+1} \, dx$

$= \int \frac{\cancel{-}(3x^2 - 2x)}{\cancel{-}(x-1)} \, dx$

$= \int \frac{3x^2 - 2x}{x-1} \, dx$

$$\begin{array}{r|l} & 3x + 1 \\ x-1 & 3x^2 - 2x \\ & -(3x^2 - 3x) \\ \hline & x \\ & -(x-1) \\ \hline & 1 \end{array}$$

$= \int (3x+1) \, dx + \int \frac{1}{x-1} \, dx$

$= \frac{3x^2}{2} + x + \ln(x-1) + C$