

NAME: AARON ABRAHAM DYEM

DEPARTMENT: COMPUTER ENGINEERING

COURSE: MAT 102

MATRIC NO: 19/Eng02/011

ASSIGNMENT

1) Find the equation of the tangent at the point $(1, 0)$ on the circle $x^2 + y^2 - 5x - y + 4 = 0$

Solution

Write Equation of the circle $= (x-a)^2 + (y-b)^2 = r^2$ using completing the square method:

$$x^2 + y^2 - 5x - y + 4 = 0$$

$$x^2 - 5x + y^2 - y = -4$$

$$(x^2 - 5x + \frac{25}{4}) + (y^2 - y + \frac{1}{4}) = -4 + \frac{25}{4} + \frac{1}{4}$$

$$(x - \frac{5}{2})^2 + (y - \frac{1}{2})^2 = \frac{5}{2}$$

Using $(x-h)^2 + (y-k)^2 = r^2$,

The centre of the circle is $(\frac{5}{2}, \frac{1}{2})$ and the radius $= \frac{\sqrt{5}}{2}$

Determine the gradient of the radius

$$m_r = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0 - \frac{1}{2}}{1 - \frac{5}{2}}$$

$$= \frac{-\frac{1}{2}}{-\frac{3}{2}}$$

$$= -\frac{1}{2} \div -\frac{3}{2}$$

$$= -\frac{1}{2} \times -\frac{2}{3}$$

$$= \frac{1}{3}$$

Determine the gradient of the tangent

$$m_r \times m_t = -1$$

$$\frac{1}{3} \times m_t = -1$$

$$\therefore m_t = -3$$

To determine the equation of the tangent to the circle,

$$m_t = -3 \text{ and } (x_1, y_1) = (1, 0)$$

$$y - y_1 = m(x - x_1)$$

$$y - 0 = -3(x - 1)$$

$$y = -3x + 3 //$$

2) Find the eqn of the tangent at the point $(1, 0)$ on the circle $x^2 + y^2 - 12x - 12y + 47 = 0$

Using completing the square

$$x^2 - 12x + y^2 - 12y = -47$$

$$(x^2 - 12x + 36) + (y^2 - 12y + 36) = -47 + 36 + 36$$

$$(x - 6)^2 + (y - 6)^2 = 25$$

The centre of the circle is $(6, 6)$ and the radius = 5

To determine the gradient of the radius

$$m_r = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0 - 6}{1 - 6}$$

$$m_r = \frac{-6}{-5} = \frac{6}{5}$$

To determine the gradient of the tangent

$$m_r \times m_t = -1$$

$$\frac{6}{5} \times m_t = -1$$

$$\therefore m_t = -\frac{5}{6}$$

To determine the equation of the tangent to the circle,

$$m_t = -\frac{5}{6} \text{ and } (x_1, y_1) = (1, 0)$$

$$y - y_1 = m(x - x_1)$$

$$y - 0 = -\frac{5}{6}(x - 1)$$

$$y = -\frac{5}{6}x + \frac{5}{6} //$$

3) Find the equation of the tangent at the point $(1, 0)$ on the circle $x^2 + y^2 - 8x + 14y + 40 = 0$

Using completing the square method;

$$x^2 - 8x + y^2 + 14y = -40$$

$$(x^2 - 8x + 16) + (y^2 + 14y + 49) = -40 + 16 + 49$$

$$(x - 4)^2 + (y + 7)^2 = 25$$

The centre of the circle = $(4, -7)$ and the radius = 5

$$m_r = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0 + 7}{1 - 4}$$

$$m_r = \frac{7}{-3} = -\frac{7}{3}$$

To determine the gradient of the tangent

$$m_r \times m_t = -1$$

$$-\frac{7}{3} \times m_t = -1$$

$$m_t = \frac{3}{7}$$

To determine the equation of the tangent to the circles,

$$m_t = \frac{3}{7} \text{ and } (x_1, y_1) = (1, 0)$$

$$y - y_1 = m_t(x - x_1)$$

$$y - 0 = \frac{3}{7}(x - 1)$$

$$y = \frac{3}{7}x - \frac{3}{7}$$