MAT 104 Assignment

Answers

Differentiation

1.) $y = \dfrac{(x+1)^2 (x-2)^{1/2}}{(2x-1)(x-3)^{4/3}}$

$\ln y = \ln[(x+1)^2] + \ln[(x-2)^{1/2}] - \ln(2x-1) - \ln[(x-3)^{4/3}]$

$\frac{1}{y}\frac{dy}{dx} = \frac{1}{(x+1)^2} \cdot 2x+2 + \frac{1}{(x-2)^{1/2}} \cdot \frac{1}{2}(x-2)^{-1/2} - \frac{1}{(2x-1)} \cdot (2) - \frac{1}{(x-3)^{4/3}} \cdot \frac{4}{3}(x-3)^{1/3}$

$\frac{1}{y}\frac{dy}{dx} = \frac{2x+2}{(x+1)^2} + \frac{1}{2}(x-2)^{-1/2} - \frac{2}{(2x-1)} - \frac{4}{3}(x-3)^{1/3}$

$\frac{1}{y}\frac{dy}{dx} = \frac{2}{(x+1)} + \frac{1}{2} - \frac{2}{(2x-1)} - \frac{4}{3}(x-3)^{1/3}$

$\frac{dy}{dx} = y\left[\frac{2}{(x+1)} + \frac{1}{2} - \frac{2}{(2x-1)} - \frac{4}{3}(x-3)^{1/3}\right]$

$\frac{dy}{dx} = \frac{(x+1)^2(x-2)^{1/2}}{(2x-1)(x-3)^{4/3}}\left[\frac{2}{(x+1)} + \frac{1}{2} - \frac{2}{(2x-1)} - \frac{4}{3}(x-3)^{1/3}\right]$

) $y = \frac{3e^x \sin 2x}{x^{5/2}}$

$y = \ln(3e^x) + \ln(\sin 2x) - \ln(x^{5/2})$

$\frac{dy}{dx} = \frac{1}{3e^x} \cdot 3e^x + \frac{1}{\sin 2x} \cdot 2\cos 2x - \frac{1}{x^{5/2}} \cdot 5x^{3/2}$

$\frac{dy}{dx} = 1 + \frac{2\cos 2x}{\sin 2x} - \frac{5x^{3/2}}{2x^{5/2}}$

$\frac{dy}{dx} = 1 + 2\tan 2x - \frac{5x^{3/2}}{2x^{5/2}}$

$\frac{dy}{dx} = y\left|1 + 2\tan 2x - \frac{5x^{3/2}}{2x^{5/2}}\right|$

$\therefore \frac{dy}{dx} = \frac{3e^x \sin 2x}{x^{5/2}}\left|1 + 2\tan 2x - \frac{5x^{3/2}}{2x^{5/2}}\right|$

Integration

1.) $\int 4\sec^2(3m+1)$

Let $u = 3m+1$

$\frac{du}{dm} = 3 \quad \therefore dx = \frac{du}{3}$

$= \int 4\sec^2 u \cdot \frac{du}{3} = \frac{1}{3}\int 4\sec^2 u\, du$

$\Rightarrow \frac{1}{3}\left|4\tan u + c\right| = \frac{4}{3}\tan u + c$

$\Rightarrow \frac{4}{3}\tan(3x+1) + c$

2.) $\int 2t(3t^2-1)^{1/2}\,dt$

Let $u = 3t^2 - 1$

$du = 6t \quad \therefore dt = \frac{du}{6t}$

dt

$\Rightarrow \int 2t \cdot u^{1/2} \cdot \frac{du}{6t} = \int u^{1/2} \frac{du}{3}$

$\frac{1}{3}\int u^{1/2}\,du \Rightarrow \frac{1}{3}\left[\frac{u^{3/2}}{3/2} + c\right]$

$\Rightarrow \frac{2u^{3/2}}{9} + c$

3.) $\int \frac{2x}{(4x^2-1)^{1/2}}\,dx$

Let $u = 4x^2 - 1$

$du = 8x \quad \therefore dx = \frac{du}{8x}$

dx

$\Rightarrow \int \frac{2x}{u^{1/2}}\,dx = \int \frac{2x}{u^{1/2}} \cdot \frac{du}{8x}$

$= \frac{1}{4}\int u^{-1/2}\,du = \frac{1}{4}\left[\frac{u^{1/2}}{1/2} + c\right.$

$= \frac{u^{1/2}}{2} + c \Rightarrow \frac{(4x^2-1)^{1/2}}{2} + c$

OR $\frac{\sqrt{(4x^2-1)}}{2} + c$