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MAT 107

Assignment

1. Find the equation of the tangent at the point (1, 0) on the circle $x^2 + y^2 - 5x - y + 4 = 0$

Solution

$(x-a)^2 + (y-b)^2 = r^2$ / $x^2 + y^2 + 2gx + 2fy + c = 0$

Centre $= (-g, -f)$

$2g = -5 \therefore g = \frac{-5}{2}$

$2f = -1 \therefore f = \frac{-1}{2}$

$\therefore$ Center $(-g, -f) = (\frac{5}{2}, \frac{1}{2})$

Radius $= \sqrt{g^2+f^2-c} = \sqrt{\left(\frac{-5}{2}\right)^2+\left(\frac{-1}{1}\right)^2-4} = \sqrt{\frac{5}{2}}$

$C=4$

Gradient of radius $= \frac{y_2-y_1}{x_2-x_1} = \frac{0-\frac{5}{2}}{1-\frac{1}{2}} = \frac{\frac{-5}{2}}{\frac{1}{2}} = \frac{-10}{2}$

$m_r = -5$

Note: The Product of the Gradient of the radius and the Gradient of the tangent line is equal to -1 i.e $m_r \times m_t = -1$

$m_t = \frac{-1}{m_r}$

∴ Therefore: $y-y_1 = m_t(x-x_1)$

Substituting for $\frac{-1}{m_r}$, $(1,0)$ we have:

$$y-0 = \frac{1}{5}(x-1)$$

$$y = \frac{1}{5}x - \frac{1}{5}$$

2. Find the equation of the tangent at the Point $(1,0)$ on the circle $x^2+y^2-12x-12y+47=0$

Solution

$x^2+y^2+2gx+2fy+c=0$

Centre $= (-g,-f)$

$2g = -12 \therefore g = -6$

$2f = -12 \therefore f = -6$

∴ Centre $(-g,-f) = (6,6)$

Radius $= \sqrt{g^2+f^2-c} = \sqrt{(-6)^2+(-6)^2-47} = \sqrt{25} = 5$

$C=47$

Gradient of radius $(m_r) = \frac{y_2-y_1}{x_2-x_1} = \frac{0-6}{1-6} = \frac{-6}{-5} = \frac{6}{5}$

$m_t = \frac{-1}{m_r} = -1 \div \frac{6}{5} = \frac{-5}{6}$

$y - y_1 = m_t(x - x_1)$

$y - 0 = \frac{-5}{6}(x - 1)$

$y = \frac{-5}{6}x + \frac{5}{6}$

3. Find the equation of the tangent at the point (1, 0) on the circle $x^2 + y^2 - 8x + 14y + 40 = 0$

Solution

$x^2 + y^2 + 2gx + 2fy + c = 0$

Center: $(-g, -f)$; $2g = -8 \therefore g = -4$

$2f = 14 \therefore f = 7$

Center: $(-g, -f) = (4, -7)$

Radius $= \sqrt{g^2 + f^2 - c} = \sqrt{(-4)^2 + (7)^2 - 40} = \sqrt{25} = 5$

$c = 40$

Gradient of radius $(m_r) = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0 + 7}{1 - 4} = \frac{7}{-3}$

$m_t = \frac{-1}{m_r} = -1 \div \frac{7}{-3} = \frac{3}{7}$

$y - y_1 = m_t(x - x_1)$

$y - 0 = \frac{3}{7}(x - 1)$

$y = \frac{3}{7}x - \frac{3}{7}$