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MAT 104

Assignment

Find the integral of the following

1. $x^2 \sin x \, dx$

Solution

$\int x^2 \sin x \, dx$

$\int u \, dv = uv - \int v \, du$

$u = x^2 \quad , \quad du = 2x \, dx$

$dv = \sin x , \quad v = -\cos x$

$= x^2 (-\cos x) - \int -\cos x (2x)$

$= -x^2 \cos x - \int -2x \cos x$

$u = -2x \quad , \quad du = -2$

$dv = \cos x \quad , \quad v = \sin x$

$-2x (\sin x) - \int \sin x (-2)$

$-2x \sin x - \int -2 \sin x$

$= -x^2 \cos x - 2x \sin x + 2 \cos x + C$

2. $3t e^{2t} \, dt$

Solution

$\int 3t \, e^{2t} \, dt$

$\int u \, dv = uv - \int v \, du$

$u = 3t \; ; \; du = 3 \, dx$

$dv = e^{2t} , \quad v = \frac{1}{2} e^{2t}$

$= 3t \left(\frac{1}{2} e^{2t}\right) - \int \frac{1}{2} e^{2t} (3)$

$= \frac{1}{2} 3t \, e^{2t} - \int \frac{3}{2} e^{2t}$

$= \frac{3}{2} t \, e^{2t} - \frac{3}{4} e^{2t} + C$

3. $2x^2 \ln x \, dx$

Solution

$2x^2 \ln x \, dx$

$\int u\,dv = uv - \int v\,du$

$u = \ln x$, $du = \frac{1}{x} dx$

$dv = 2x^2$, $v = \frac{2x^3}{3}$

$= \ln x\left(\frac{2x^3}{3}\right) - \int \frac{2x^3}{3}\left(\frac{1}{x}\right)$

$= \frac{2x^3 \ln x}{3} - \int \frac{2x^3}{3x}$

$= \frac{2x^3}{3} \ln x - \frac{2x^3}{9x} + C$

$= \cancel{\frac{2x^3}{3}\left[x^3 \ln x - \frac{x^4}{4x}\right]} + C$ $\quad \frac{2}{3} x^3 \left[\ln x - \frac{1}{3}\right] + C$

4. $\frac{(2x - 3x^2)}{(1-x)} dx$

Solution

$\cancel{3x^2}$

$1 - x \,\overline{)\, -3x^2 + 2x}$

Rewrite: $\int \frac{3x^2 - 2x}{x-1} dx$

~~Substitute $u = x - 1$~~ $\rightarrow \frac{du}{dx} = 1$ $\quad \int \left(3x + \frac{1}{(x-1)} + 1\right) dx$

~~$dx = du$, use:~~

~~$x^2 = (u+1)^2$~~ $\quad$ From long division

$= \int \cancel{\frac{3u^2 + 4u + 1}{u} du}$

Apply linearity:

$= 3\int x\,dx + \int \frac{1}{x-1} dx + \int 1\,dx$

Now solving

$\int x\,dx$

Apply Power rule!

$$\int x^n dx = \frac{x^{n+1}}{n+1} \quad \text{with } n=1$$

$$= \frac{x^2}{2}$$

Now solving

$$\int \frac{1}{x-1} dx$$

Substitute $u = x-1 \rightarrow \frac{du}{dx} = 1$

$$dx = dx$$

$$= \int \frac{1}{u} du$$

This is a standard integral

$$= \ln(u)$$

Undo substitution $u = x-1$

$$= \ln(x-1)$$

Now solving!

$$\int 1 dx$$

Apply Constant rule

$$= x$$

Plug in solved integrals

$$3\int x dx + \int \frac{1}{x-1} dx + \int 1 dx$$

$$= \frac{3x^2}{2} + x + \ln(x-1)$$

$$\int \frac{2x-3x^2}{1-x} dx = \frac{3x^2}{2} + x + \ln(|x-1|) + C$$