

Math 104

$$1. \int x^2 \sin x \, dx$$

Solution

$$\int u \, dv = uv - \int v \, du \quad (1)$$

$$\text{For } \int u \, dv = \int x^2 \sin(x) \, dx$$

$$u = x^2 \Rightarrow \frac{du}{dx} = 2x$$

$$du = 2x \, dx$$

$$dv = \sin(x) \, dx$$

$$\int dv = \int \sin(x) \, dx$$

$$v = -\cos(x)$$

Substitute in eqn (1)

$$\int x^2 \sin(x) \, dx = -x^2 \cos(x) - \int (-2x \cos(x)) \, dx$$

$$\int x^2 \sin(x) \, dx = -x^2 \cos(x) + 2 \int x \cos(x) \, dx \quad (2)$$

$$u = x \Rightarrow \frac{du}{dx} = 1 \Rightarrow du = dx$$

$$dv = \cos(x) \, dx \Rightarrow \int dv = \int \cos(x) \, dx$$

$$v = \sin(x)$$

$$\int x \cos(x) \, dx = x \sin(x) - \int \sin(x) \, dx$$

Since $\int \sin(x) \, dx = -\cos(x)$, this becomes

$$\int x \cos(x) \, dx = x \sin(x) + \cos(x) \dots (3)$$

Now, returning to before.

$$\int x^2 \sin(x) dx = -x^2 \cos(x) + 2 \int x \cos(x) dx$$

Substitute in (3) into (2)

$$\int x^2 \sin(x) dx = -x^2 \cos(x) + 2(x \sin(x) + \cos(x))$$

$$\int x^2 \sin(x) dx = -x^2 \cos(x) + 2x \sin(x) + 2 \cos(x) + C$$

2. $\int 3t e^{2t} dt$

$$\int u dv = uv - \int v du$$

Let $u = 3t$

$$\frac{du}{dt} = 3, \text{ so } du = 3 dt$$

Let $dv = e^{2t} dt$

$$v = \frac{1}{2} e^{2t}$$

$$\begin{aligned} \int 3t(e^{2t}) dt &= 3t \left(\frac{1}{2}\right) e^{2t} - \int \frac{1}{2} e^{2t} 3 dt \\ &= \frac{3}{2} t e^{2t} - \frac{3}{4} e^{2t} \end{aligned}$$

3. $\int 2x^2 \times \ln x dx$

(Commutative property)

$$2 \times \int x^2 \ln(x) dx$$

↓

$$2 \times \ln(x) \times x^2 dx$$

$$\int u dv = uv - \int v du$$

$$u = \ln(x)$$

$$dv = x^2 dx$$

$$v = \int dv$$

$$du = \frac{1}{x} dx$$

$$v = x^3/3$$

$$2 \left(\ln(x) \times \frac{x^3}{3} - \int \frac{x^3}{3} \times \frac{1}{x} dx \right)$$

$$\int u dv = uv - \int v du$$

$$\int 2x^2 \ln x dx = 2 \left(\ln x \times \frac{x^3}{3} \right) - \int \frac{x^3}{3} \times \frac{1}{x} dx$$

$$2x^2 \ln x dx = 2 \left(\ln x \times \frac{x^3}{3} - \int \frac{x^2}{3} dx \right)$$

$$= 2 \left(\ln x \times \frac{x^3}{3} - \frac{1}{3} x \left(\int x^2 dx \right) \right)$$

$$\int x^n dx = \frac{x^{n+1}}{n+1}$$

$$\int x^2 dx = \frac{x^{2+1}}{2+1} = \frac{x^3}{3}$$

$$= 2 \left(\ln(x) \times \frac{x^3}{3} - \frac{1}{3} \times \frac{x^3}{3} \right)$$

$$= \frac{2x^3 \times \ln(x)}{3} - \frac{2x^3}{9}$$

$$\int 2x^2 \ln x dx = \frac{2x^3 \times \ln x}{3} - \frac{2x^3}{9} + c$$

$$+ \int \frac{2x - 3x^2}{1-x} dx, \text{ Separate} = \int \frac{2x}{1-x} dx - \int \frac{3x^2}{1-x} dx$$

Property of Integral

$$\int f(x) \pm g(x) dx = \int f(x) dx \pm \int g(x) dx$$

$$\int \frac{2x}{1-x} dx - \int \frac{3x^2}{1-x} dx$$

(calculate indefinite integrals)

$$2 - 2x - 2 \ln|1-x| + \frac{-9 + 6x + 3x^2}{2} + 3 \ln|1-x|$$

Simplify

$$\frac{-5 + 2x + 3x^2}{2} + \ln|1-x|$$

$$\therefore \int \frac{2x - 3x^2}{1-x} = \frac{-5 + 2x + 3x^2}{2} + \ln|1-x|$$