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19 | ENG05 | D48

MECHATRONICS ENGINEERING

MAT 102 ASSIGNMENT

~~Elizah~~

(1) Find the equation of the tangent at the point (x_2, y_2) on the circle $x^2 + y^2 - 5x - 4y + 4 = 0$

$$\therefore x^2 + y^2 - 5x - 4y + 4 = 0$$

$$x^2 - 5x + y^2 - 4y = -4$$

$$x^2 - 5x + (-5/2)^2 + y^2 - 4y + (-1/2)^2 = -4 + (-5/2)^2 + (-1/2)^2$$

$$(x - 5/2)^2 + (y - 1/2)^2 = -4 + 25/4 + 1/4$$

$$(x - 5/2)^2 + (y - 1/2)^2 = \frac{-16 + 25 + 1}{4}$$

$$(x - 5/2)^2 + (y - 1/2)^2 = 10/4$$

\therefore centers $(a, b) = (5/2, 1/2)$ conforming with equation of

a circle : $(x - a)^2 + (y - b)^2 = r^2$

$$\therefore r^2 = 10/4; \therefore r = \sqrt{10/4} = \sqrt{10}/2$$

$$\text{Gradient of radius} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0 - 1/2}{1 - 5/2} = \frac{-1/2}{-3/2} = \frac{1}{3}$$

$$\therefore \text{Gradient of Radius } (m_1) = 1/3$$

Radius \perp Tangent; $\therefore M_1 M_2 = -1$

$$M_2 (\text{Gradient of Tangent}) \therefore M_2 = -1/m_1 = -1/(1/3) = -3$$

$$\therefore \text{Equation of Tangent} = y - y_1 = m(x - x_1)$$

$$\therefore y - 0 = -3(x - 1)$$

$$\therefore y = -3x + 3$$

Equation of Tangent.

(2) Find the equation of tangent: points (1,0) equation on circle:

$$x^2 + y^2 - 12x - 12y + 47 = 0$$

$$\therefore x^2 - 12x + y^2 - 12y = -47$$

$$\therefore x^2 - 12x + (-6)^2 + y^2 - 12y + (-6)^2 = -47 + (-6)^2 + (-6)^2$$

$$(x-6)^2 + (y-6)^2 = -47 + 72$$

$$(x-6)^2 + (y-6)^2 = 25$$

\therefore Co-ordinate of centre (a,b) is (6,6) and $r^2 = 25$; $\therefore r = 5$

\therefore radius = 5

$$\text{Gradient of Radius } (M_1) = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0 - 6}{1 - 6} = \frac{+6}{+5} = \frac{6}{5}$$

$$M_1 M_2 = -1; \therefore M_2 = -\frac{1}{6/5} = -\frac{5}{6}$$

\therefore Gradient of Tangent = $-\frac{5}{6}$

\therefore Equation of Tangent; $y - y_1 = m(x - x_1)$

$$y - 0 = -\frac{5}{6}(x - 1)$$

$$\therefore \frac{6y}{6} = \frac{-5x + 5}{6}$$

$$\therefore y = -\frac{5}{6}x + \frac{5}{6}$$

Equation of Tangent.

(3) Find the equation of tangent: point (1,0) equation on circle:

$$x^2 + y^2 - 8x + 14y + 40 = 0$$

$$x^2 - 8x + y^2 + 14y = -40$$

$$x^2 - 8x + (-4)^2 + y^2 + 14y + (7)^2 = -40 + (-4)^2 + 7^2$$

$$(x-4)^2 + (y+7)^2 = -40 + 16 + 49$$

$$(x-4)^2 + (y+7)^2 = 18$$

\therefore Co-ordinate of centre (a,b) is (4,-7); $r^2 = 18$ \therefore radius = $\sqrt{18}$

$$M_1 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0 - 4}{1 - (-7)} = \frac{-4}{8} = -\frac{1}{2}$$

$$M_1 M_2 = -1$$

$$\therefore M_2 = -1/m_1 = -1/1/2 = 2$$

\therefore Gradient of Tangent = 2

$$\therefore \text{Equation of Tangent} : y - y_1 = m(x - x_1)$$

$$y - 0 = 2(x - 1)$$

$$\boxed{\therefore y = 2x - 2}$$

Equation of Tangent