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MAT 104 Assignment

(1) $\int \frac{1}{\sqrt{4-x^2}}$ ($x = 2\sin\theta$)

solution

Recall,

$\int \frac{1}{\sqrt{4-x^2}} dx$

$\int \frac{1}{\sqrt{2^2-x^2}} dx$

let $x = 2\sin\theta$

$\frac{dx}{d\theta} = 2\cos\theta$

$dx = 2\cos\theta d\theta$

$2^2 - x^2 = 2^2 - 2^2\sin^2\theta$

$= 2^2(1 - \sin^2\theta)$

$= 2^2(\cos^2\theta)$

$= 4\cos^2\theta$

$\int \frac{dx}{\sqrt{4-x^2}} = \int \frac{2\cos\theta\, d\theta}{\sqrt{4\cos^2\theta}} = \int \frac{2\cos\theta\, d\theta}{2\cos\theta} = \int \frac{2\cos\theta\, d\theta}{2\cos\theta}$

$= \int d\theta = \theta + C$

$= \sin^{-1}\frac{x}{2} + C_{11}$

Recall, $x = 2\sin\theta$

$\therefore \theta = \sin^{-1}\frac{x}{2}$

(2) $\int \frac{1}{\sqrt{4-x^2}}$

solution

Square the fraction to cancel out the square root

$\frac{1^2}{(\sqrt{4+x^2})^2} = \frac{1}{4+x^2}$

$$\int \frac{1}{4+x^2}\,dx = \int \frac{1}{2^2+x^2}\,dx$$

$x = 2\tan\theta$

$\frac{dx}{d\theta} = 2\sec^2\theta$

$dx = 2\sec^2\theta\,d\theta$

$2^2+x^2 = 2\sec^2\theta$

$$\int \frac{2\sec^2\theta\,d\theta}{2\sec^2\theta} = \frac{1}{2}\int d\theta = \frac{1}{2}[\theta] + c$$

Recall $x = 2\tan\theta$

$\theta = \tan^{-1}\frac{x}{2}$

$$\therefore \frac{1}{2}\int d\theta = \frac{1}{2}\tan^{-1}\frac{x}{2} + c$$