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MECHATRONICS ENGINEERING
19/ENG05/043

1) $x^2 + y^2 - 5x - y + 4 = 0$, $(1,0)$

$x^2 + y^2 - 5x - y = -4$

$x^2 - 5x + (5/2)^2 + y^2 - y + (1/2)^2 = -4 + 25/4 + 1/4$

$= (x + 5/2)^2 + (y + 1/2)^2 = 6\,1/2 - 4$

$= (x + 5/2)^2 + (y + 1/2)^2 = 2\,1/2$.

Centre $= (-5/2, -1/2)$.

$(\text{radius})^2 = 2.5$ or $2\,1/2$ $\therefore$ radius $= \sqrt{2.5} = 0.5$

Gradient of the radius $= \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{0 + 1/2}{1 + 5/2} = \dfrac{1/2}{7/2}$

$= 1/2 \times 2/7 = 1/7$

Gradient of the tangent $= 1/7 \times m = -1$

$\therefore m = -7$

Equation of tangent to the circle $= y - y_1 = m(x - x_1)$

$= y - y_1 = -7(x - x_1)$

Sub $P(1,0) = y - 0 = -7(x - 1)$

$y = 0 \therefore 0 = -7x + 7$.

$y = -7x + 7$

2) $x^2 + y^2 - 12x - 12y + 47 = 0$

$x^2 + y^2 - 12x - 12y = -47$

$x^2 - 12x + (6)^2 + y^2 - 12y + (6)^2 = -47 + 36 + 36$

$(x + 6)^2 + (y + 6)^2 = 25$

Centre $= (-6, -6)$

$(\text{radius})^2 = 25$ $\therefore$ radius $= \sqrt{25} = 5$

Gradient of the radius $= \frac{y_2 - y_1}{x_2 - x_1} = \frac{0+6}{1+6} = \frac{6}{7}$.

Gradient of the tangent $= \frac{6}{7} \quad m = -1 = \frac{6m}{7} = -1$

~~$m = -\frac{6}{7}$~~ $\quad m = -\frac{7}{6}$

Equation of tangent to the circle $= y - y_1 = m(x - x_1)$.

$y - y_1 = -\frac{7}{6}(x - x_1)$

Sub $P(1,0) = y - 0 = -\frac{7}{6}(x - 1)$

$y = -\frac{7}{6}x + \frac{7}{6}$

3) $x^2 + y^2 - 8x + 14y + 40 = 0$

$x^2 + y^2 - 8x + 14y = -40$

$x^2 - 8x + 4^2 + y^2 + 14y + 7^2 = -40 + 16 + 49$

$= (x+4)^2 + (y-7)^2 = 25$

Centre $= (-4, 7)$

$(\text{radius})^2 = 25 \quad \therefore \text{radius} = \sqrt{25} = 5$

Gradient of the radius $= \frac{y_2 - y_1}{x_2 - x_1} = \frac{0-7}{1+4} = -\frac{7}{5}$

Gradient of the tangent $= -\frac{7}{5} \quad m = -1$

$m = \frac{5}{7}$

Equation of tangent to the circle $= y - y_1 = m(x - x_1)$

Sub $P(1,0) \therefore y - 0 = \frac{5}{7}(x - 1)$.

$y = \frac{5x}{7} - \frac{5x}{7}$