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MATRICULATION NUMBER : 19/MHS01/089
COURSE : MAT 104 (GENERAL MATHEMATICS III)

1: $\int \frac{2x}{\sqrt{4x^2-1}} \, dx$

SOLUTION:

$$\int \frac{2x}{\sqrt{4x^2-1}} = \int 2x \times \frac{1}{\sqrt{4x^2-1}}$$

let $u = 4x^2 - 1$

$$\frac{du}{dx} = 8x$$

$$\frac{dx}{du} = \frac{1}{8x}$$

$$\int 2x \times \frac{1}{u} \, du$$

$$\frac{dx}{du} \Big| \; 2x \times \frac{1}{u^{1/2}} = \frac{1}{8x} \Big| \; 2x \times \frac{u^{-1/2+1}}{-1/2+1}$$

$$= \frac{1}{8x} \Big| \; 2x \times \frac{u^{1/2}}{}$$

$$= \frac{1}{2} \times \frac{u^{1/2}}{1}$$

$$= \frac{1}{2} \times u^{1/2}$$

Recall $u^2 = 4x^2 - 1$

$$\frac{1}{2}\left(\sqrt{4x^2-1}\right) + C$$

2 $\int \frac{\sin^{-1} x}{\sqrt{1-x^2}} \, dx$

SOLUTION:

Recall $\int a f(x) [ f^2 (x))$

$$= \frac{a(fx)^2}{2} + c$$

That is $\frac{1}{\sqrt{1-x^2}}$ is a derivative of $\sin^{-1} x$

$\int \frac{\sin^{-1} x}{\sqrt{1-x^2}}$ can also be written as $\int \sin^{-1} x \times \frac{1}{\sqrt{1-x^2}}$

let $u = 1 - x$

$\frac{du}{dx} = -1$ , $\frac{dx}{du} = -1$

$\frac{du}{dx} \Big| \sin^{-1} x \times u^{1/4} = g(x) = \sin^{-1}$

$g'(x) = \frac{1}{\sqrt{1-x^2}}$

$u \times du = \frac{u^{1+1}}{1+1}$

$\frac{u^2}{2} + c$

Recall $u = \sin^{-1} x$

$$\frac{(\sin^{-1} x)^2}{2} + c$$

3. $\int (\tan x)^6 \sec^2 x \, dx$

SOLUTION ;

$u = \tan x$

$du = \sec^2 x \, dx$

$u^6 du = u$

$f'(x) = (\tan x)^6$

$f(x) = \sec^2 x$

$u^6 \times du = \dfrac{u^{6+1}}{6+1}$

$u^6 \times du = \dfrac{u^7}{7} + c$

$\therefore u^6 du = \dfrac{(\tan x)^7}{7} + c$