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Assignment

1) Find the equation of the tangent of the point $(1, 0)$

on the circle $x^2 + y^2 - 5x - y + 4 = 0$

Solution:

$$x^2 + y^2 - 5x - y + 4 = 0$$

Comparing the given equation to $x^2 + y^2 + 2gx + 2fy + c = 0$

$$\frac{2gx}{2x} = \frac{-5x}{2x}$$

$$g = \frac{-5}{2}$$

$$\frac{2fy}{2y} = \frac{-1}{2y}$$

$$f = \frac{-1}{2}$$

$$(x_1, y_1) = (1, 0)$$

Using equation of the tangent at point $(1, 0)$

$$y - y_1 = m(x - x_1)$$

$$\text{where } m = \frac{-(x_1 + g)}{y_1 + f}$$

$$y - y_1 = \left[\frac{-(x_1 + g)}{y_1 + f} \right] (x - x_1)$$

$$y - 0 = \left[\frac{-(1 + (-5/2))}{0 + (-1/2)} \right] (x - 1)$$

$$y = \left[\frac{-(1 - 5/2)}{-1/2} \right] (x - 1)$$

$$y = \left[\frac{-(1 - 5)}{1 - 2} \div \frac{-1}{2} \right] (x - 1)$$

$$y = \left[\frac{-(2 - 5)}{2} \times \frac{2}{-1} \right]$$

$$y = -3(x - 1)$$

$$y = -3x + 3$$

$$y + 3x - 3 = 0$$

∴ Equation of the tangent = $y + 3x - 3 = 0$

(2) Find the equation of the tangent at the point (1, 0) on the circle $x^2 + y^2 - 12x - 12y + 47 = 0$

Solution:

$$x^2 + y^2 - 12x - 12y + 47 = 0$$

Comparing the given equation to $x^2 + y^2 + 2gx + 2fy + c = 0$

$$\frac{2g}{2x} = \frac{12x}{2x}$$

$$g = -6$$

$$\frac{2f}{2y} = \frac{-12y}{2y}$$

$$F = -6$$

$$C(x_1, y_1) = (1, 0)$$

using equation of the tangent at point $C(x_1, y_1)$

$$y - y_1 = m(x - x_1)$$

$$\text{where } m = -\frac{C(x_1, y_1)}{y_1 + F}$$

$$m = -\frac{C(1 + (-6))}{(0 + (-6))}$$

$$m = \frac{-(-5)}{6}$$

$$m = \frac{5}{6}$$

$$m = \frac{5}{6}$$

$$y - 0 = \frac{5}{6}(x - 1)$$

$$y = \frac{5x}{6} + \frac{5}{6} = \frac{5x + 5}{6}$$

$$y = \frac{5x + 5}{6}$$

$$5y = 5x + 5$$

$$5y + 50x - 5 = 0$$

\therefore Equation of the tangent = $5y + 50x - 5 = 0$

③ Find the equation of the tangent at the point $(4, 0)$

on the circle $x^2 + y^2 - 8x + 14y + 40 = 0$

Comparing the given equation to $x^2 + y^2 + 2gx + 2fy + c = 0$

Solution:

$$x^2 + y^2 - 8x + 14y + 40 = 0$$

Comparing the given equation to $x^2 + y^2 + 2gx + 2fy + c = 0$

$$\frac{2gx}{2x} = \frac{-8x}{2x}$$

$$g = -4$$

$$\frac{2fy}{2y} = \frac{14y}{2y}$$

$$f = 7$$

$$(x_1, y_1) = (4, 0)$$

using equation of the tangent at point (x_1, y_1)

$$y - y_1 = m(x - x_1)$$

where $m = \frac{-f(x_1 + g)}$

$$m = \frac{-f(x_1 + g)}{y_1 + f} = \frac{-7(4 - 4)}{0 + 7} = \frac{-7(-3)}{7} = 3$$

$$\frac{3}{7}$$

$$m = \frac{3}{7}$$

$$y - (0) = \frac{3}{7} (x - 1)$$

$$y = \frac{3x}{7} - \frac{3}{7} = \frac{3x - 3}{7}$$

$$y = \frac{3x - 3}{7} \Rightarrow 7y = 3x - 3$$

∴ Equation of the tangent = $7y - 3x + 3 = 0$