NAME : AKUKWE MICHAEL LOTACHUKWU

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Question 1
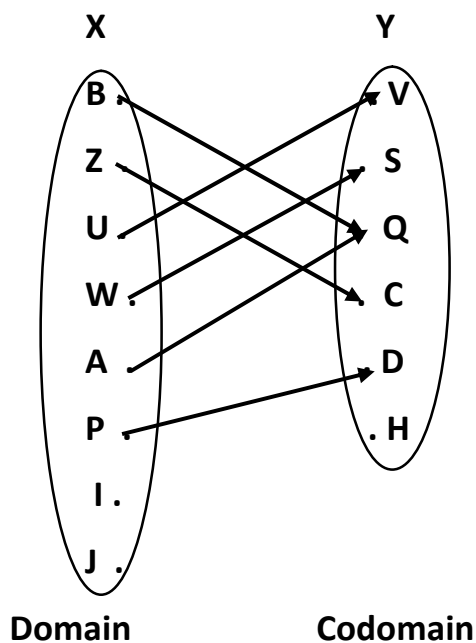
Linear transformation T: X ⟶ Y is a function that carries elements of the vector space X (domain) to the vector space V (codomain) which has 2 properties:

Additive property: $T(U_1 + U_2) = T(U_1) + T(U_2)$

Multiplicative property: $T(\alpha U) = \alpha T(U)$

Forexample

1. Considering the 2 vector spaces X and Y below. Where X is the domain containing 8 elements and Y is the codomain containing 6.

Domain   Codomain

T : X ⟶ Y

T(B) = T(A) = Q

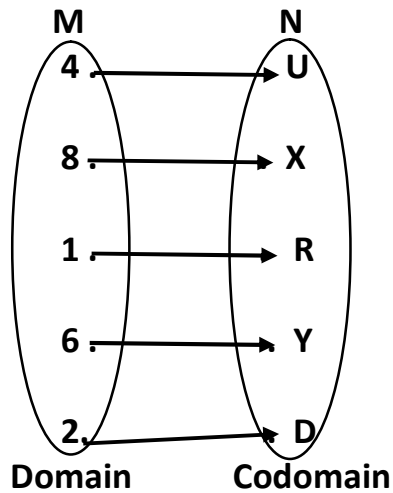T(Z) = C

T(U) = V

T(W) = S

T(P) = D

2. Also considering 2 vector spaces M and N containing 5 elements each, where M is the domain and N is the codomain

M   **N**

4 → U

8 → X

1 → R

6 → Y

2 → D

Domain   **Codomain**

T : M ⟶ N

T(4) = U

T(8) = X

T(1) = R

T(6) = Y

T(2) = D

EXPLANATION

In the examples above, the ovals represents the vector spaces and a small dot inside the ovals represents the vectors. The oval on the left represents the domain and that on the right represents the codomain. To convey that the linear transformation associates a certain input with a certain output we draw an arrow from the input to the output.

 In the first example, we see that a vector space can have more elements than the other vector space and we also see that it is possible for one or more inputs not to be associated with any output. And it is also possible for one or more inputs to be associated with one output.

And in the second example, we see that it is also possible for all inputs to be associated with all outputs and the number of elements in both ovals to be equal.

Question 2

T(X) = AX

$$T(X) = \begin{pmatrix} 1 & 9 & 3 \\ -2 & 6 & 7 \\ 0 & -1 & 3 \end{pmatrix} \begin{pmatrix} 1 \\ 4 \\ -8 \end{pmatrix}$$

$$T(X) = 1\begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix} + 4\begin{bmatrix} 9 \\ 6 \\ -1 \end{bmatrix} + (-8)\begin{bmatrix} 3 \\ 7 \\ 3 \end{bmatrix}$$

$$T(X) = \begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix} + \begin{bmatrix} 36 \\ 24 \\ -4 \end{bmatrix} + \begin{bmatrix} -24 \\ -56 \\ -24 \end{bmatrix}$$

$$T(X) = \begin{bmatrix} 13 \\ -34 \\ -28 \end{bmatrix}$$

Hence, transformation of $\begin{bmatrix} 1 \\ 4 \\ -8 \end{bmatrix}$ gives $\begin{bmatrix} 13 \\ -34 \\ -28 \end{bmatrix}$

Question 3

Rank of a matrix is the dimension of the vector space generated by it's columns. It is also the maximum number of linearly independent column vectors in the matrix. It is the dimension of the row space of a matrix. It is denoted by R(A).

Forexample

1. Consider the matrices below:

i) $\quad A = \begin{pmatrix} 1 & -3 & 6 \\ 4 & 0 & 2 \\ 8 & 5 & 1 \end{pmatrix}$
   ii) $\quad B = \begin{pmatrix} 3 & 9 & 2 \\ 1 & 5 & 6 \\ 2 & 7 & 4 \end{pmatrix}$

i)   Rank of A

$|A| = \begin{vmatrix} 1 & -3 & 6 \\ 4 & 0 & 2 \\ 8 & 5 & 1 \end{vmatrix}$

$|A| = 1\begin{vmatrix} 0 & 2 \\ 5 & 1 \end{vmatrix} - (-3)\begin{vmatrix} 4 & 2 \\ 8 & 1 \end{vmatrix} + 6\begin{vmatrix} 4 & 0 \\ 8 & 5 \end{vmatrix}$

$|A| = 1[0 - 10] + 3[4 - 16] + 6[20 - 0]$

$|A| = -10 - 36 + 120$

$|A| = 74$

Since $|A| \neq 0$, the rank of the matrix A is 3.

ii)   Rank of B

$|B| = \begin{vmatrix} 3 & 9 & 2 \\ 1 & 5 & 6 \\ 2 & 7 & 4 \end{vmatrix}$

$|B| = 3\begin{vmatrix} 5 & 6 \\ 7 & 4 \end{vmatrix} - 9\begin{vmatrix} 1 & 6 \\ 2 & 4 \end{vmatrix} + 2\begin{vmatrix} 1 & 5 \\ 2 & 7 \end{vmatrix}$

$|B| = 3[20 - 42] - 9[4 - 12] + 2[7 - 10]$

$|B| = -66 + 72 - 6$

$|B| = 0$

Since $|B| = 0$ then we delete a row and a column in the matrix B

So, deleting the 1st row and the 2nd column of the matrix B

$B = \begin{pmatrix} 1 & 6 \\ 2 & 4 \end{pmatrix}$

/B/ = $\begin{vmatrix} 1 & 6 \\ 2 & 4 \end{vmatrix}$

/B/ = [4 − 12]

/B/ = -8

Since **/B/ ≠ 0**, the rank of the matrix is 2

Explanation

To determine the rank of a matrix we have to calculate the determinant of the matrix and the matrix has to be a square matrix (e.g 2 x 2, 3 x 3, 4x4, etc) if not the rank of that matrix cannot be determined. For a rank of a matrix to be known the determinant must not be equal to zero (0) and if the determinant of the matrix is equal to zero then we delete any row and column of our choice, like we did in the second example.

Looking at the first example, we can see that the determinant of the 3 x 3 matrix is equal to 74 (i.e it is not equal to zero), therefore the rank of the matrix is 3 and in the second example the determinant of the 3 x 3 matrix is zero, so we deleted the 1st row and 2nd column thereby reducing it to a 2 x 2 matrix and we got -8 as the determinant so the rank of the matrix is 2.