

Math 102

Mechanical Engineering

$$(1) x^2 + y^2 - 5x - y + 4 = 0 \quad \text{at } (1, 0) = (x_1, y_1)$$

$$\text{Let } 2gx = -5x, \quad g = -\frac{5}{2}, \quad a = \frac{5}{2}$$

$$\text{Let } 2fy = -y, \quad f = -\frac{1}{2}, \quad b = \frac{1}{2}$$

$$\text{Gradient, } m_T = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{0 - \frac{5}{2}}{1 - \frac{1}{2}} = \frac{-\frac{5}{2}}{\frac{1}{2}} = \frac{-\frac{5}{2} \cdot 2}{\frac{1}{2} \cdot 2} = \frac{-5}{1} = -5$$

$$\text{For perpendicularity, } m_T = \frac{1}{5}$$

Now equation of Tangent

$$\Rightarrow y - y_1 = m_T (x - x_1)$$

$$\Rightarrow y - 0 = \frac{1}{5} (x - 1)$$

$$\Rightarrow 5(y) = 1(x - 1)$$

$$5y = x - 1$$

$$5y - x + 1 = 0$$

(This is the required equation)

Note: For perpendicularity $m_1 m_2 = -1$, i.e., $-5 \times \frac{1}{5} = -1$

$$(2) x^2 + y^2 - 12x - 12y + 47 = 0 \quad \text{at } (1, 0)$$

$$\text{Let } 2gx = -12x, \quad g = -6, \quad a = 6$$

$$\text{Let } 2fy = -12y, \quad f = -6, \quad b = 6$$

$$\text{Gradient, } m_T = \frac{y_2 - y_1}{x_2 - x_1}, \quad \frac{0 - 6}{1 - 6} = \frac{-6}{-5} = \frac{6}{5}$$

$$\therefore \text{For perpendicularity, } m_T = -\frac{5}{6}$$

Now equation of tangent

$$\Rightarrow y - y_1 = m_T (x - x_1)$$

$$\Rightarrow y - 0 = -\frac{5}{6} (x - 1)$$

$$6y = -5x + 5$$

$$6y + 5x - 5 = 0 \quad (\text{This is the required equation})$$

$$(3) \quad x^2 + y^2 - 8x + 14y + 40 = 0 \text{ at } (1, 0)$$

Centre of the circle

$$(4, -7)$$

Hence, the gradient is $\frac{0 - (-7)}{1 - 4} = \frac{7}{-3}$

\therefore The tangent has a gradient of $\frac{3}{7}$ so, its equation is:

$$y - 0 = \frac{3}{7}(x - 1)$$

$$7y - 3x + 3 = 0 \quad \left(\begin{array}{l} \text{The required Equation} \\ \text{of the tangential line to} \\ \text{the circle} \end{array} \right)$$

M