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1) Find the equation of the ~~equation~~ tangent at the point (1,0) on the circle $x^2 + y^2 - 5x - y + 4 = 0$

soln

Rewriting the eqn: $x^2 - 5x + y^2 - y = -5 \quad -(1)$

Using completing the square method for ~~eqn (1)~~ $x^2 - 5x$

from $ax^2 + bx + c = 0$ where $a = 1, b = -5, c = 0$

$a(x+d)^2 + e$ (considering vertex form of a parabola)

$d = b/2a = -5/2(1) = -5/2 = -2.5$

$e = c - (b^2/4a) = 0 - ((-5)^2/4(1)) = -25/4 = -6.25$

$a(x+d)^2 + e = (x + (-2.5))^2 + (-6.25)$

substitute $(x - 2.5)^2 - 6.25$ for $x - 5x$ in eqn (1)

$(x - 2.5)^2 - 6.25 + y^2 - y = -4$

$(x - 2.5)^2 + y^2 - y = +6.25 - 4$

completing the square method for $y^2 - y$

$a(y+d)^2 + e$. $\quad a = 1, b = -1, c = 0$

$d = b/2a = -1/2(1) = -1/2 = -0.5$

$e = c - (b^2/4a) = 0 - ((-1)^2/4(1)) = -1/4 = -0.25 \Rightarrow (y - 0.5) - 0.25$

$(x - 2.5)^2 + (y - 0.5) = -4 + 6.25 + 0.25$

$(x - 2.5)^2 + (y - 0.5) = 2.5$

From eqn of a circle: $(x - h)^2 + (y - k)^2 = r^2$

$r = \sqrt{2.5}, \; h = 2.5, \; k = 0.5$

The centre is at $(h, k) = (2.5, 0.5)$

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(1) (continues)

from $(1,0)$ and $(2.5, 0.5)$ $\quad y_2 = 0.5, y_1 = 0, x_2 = 2.5, x_1 = 1$

$$\text{slope } m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0.5 - 0}{2.5 - 1} = \frac{0.5}{1.5}$$

$$\text{Tangent slope, } m_T = -\frac{1}{m} = \frac{-1.5}{0.5} = \underline{\underline{-3}}$$

$y = -3x + b$

$b = y + 3x$

$b = 0 + 3(1) = 3$

$y = -3x + 3$

eqn of tangent $\Rightarrow 3x + y - 3 = 0$

2) Find the equation of the tangent at the ~~tang~~ point $(1,0)$ on the circle $x^2 + y^2 - 12x - 12y + 47 = 0$

soln

$x^2 - 12x + y^2 - 12y = -47 \quad -(i)$

Using completing the square method for eqn (i)

$\cancel{(x-6)^2 + (y+6)^2 = 22}$

$(x-6)^2 + (y-6)^2 = 25$

From eqn of a circle: $(x-h)^2 + (y-k)^2 = r^2$

$r = 5, \quad h = 6, \quad k = 6$

The centre is at $(h, k) = (6, 6)$

from $(1,0)$ and $(6,6)$ $\quad y_2 = 6, y_1 = 0, x_2 = 6, x_1 = 1$

$$\text{slope } m_1 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6 - 0}{6 - 1} = \frac{6}{5}$$

Tangent slope $m_T = -1/m = -5/6$

$y = -5/6\, x + b$

$b = y + (-5/6\, x)$

(2) continuation.

$b = 0 + 5/6 (1)$

$b = 5/6$

$y = -\frac{5}{6}x + \frac{5}{6}$

$6y = -5x - 5 \Rightarrow -5x - 6y - 5$ multiply by -1

eqn of tangent $\Rightarrow 5x + 6y + 5$.

3) Find the equation of the tangent at the point $(1, 0)$ on the circle $x^2 + y^2 - 8x + 14y + 40 = 0$

Sln.

$x^2 - 8x + y^2 + 14y = -40 \quad —(1)$.

Using completing the square method for eqn. (1)

$(x-4)^2 + (y+7)^2 = 49 + 16 - 40$

$(x-4)^2 + (y+7)^2 = 25$

from eqn of a circle : $(x-h)^2 + (y-k)^2 = r^2$

$r = 5, \ h = 4, \ k = -7$

The centre of the circle is at : $(h, k) = (4, -7)$

from $(1, 0)$ and $(4, -7)$ where $y_2 = -7, y_1 = 0, x_2 = 4, x_1 = 1$

Slope $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-7 - 0}{4 - 1} = \frac{-7}{3}$

Tangent slope $M_T = -(1/m) = -(-3/7) = 3/7$

$y = 3/7 x + b$

$b = y - 3/7 x$

$b = 0 - 3/7 (1)$

$b = -3/7$

$y = 3/7 x - 3/7$

$\cancel{7y = 3/7x} - \cancel{3} \quad 7y = +3x - 3$

equation of tangent $\Rightarrow 3x - 7y - 3$