INYANG MARTIN VICTOR 19/SCI01/053
COMPUTER SCIENCE

1 $\int x^2 \sin x \, dx$

Solution

$$\int u\,dv = uv - \int v\,dv \quad \text{—} (1)$$

for $\int u\,dv = \int x^2 \sin(x)\,dx$

$u = x^2 \Rightarrow \frac{du}{dx} = 2x$

$du = 2x\,dx$

$dv = \sin(x)\,dx$

$v = -\cos x$

Substitute into eq (1)

$$\int x^2 \sin(x)\,dx = -x^2\cos(x) - \int(-2x\cos(x))\,dx$$

$$\int x^2 \sin(x)\,dx = -x^2\cos(x) + 2\int x\cos(x)\,dx \quad \text{—} (2)$$

$v = x \Rightarrow \frac{du}{dx} = 1 \Rightarrow du = dx$

$dv = \cos(x)\,dx \Rightarrow \int dv = \int \cos(x)\,dx$

$v = \sin(x)$

$$\int x\cos(x)\,dx = x\sin(x) - \int \sin(x)\,dx$$

Since $\int \sin(x)\,dx = -\cos(x)$, this becomes

$$\int x\cos(x)\,dx = x\sin(x) + \cos(x) \quad \text{—} (3)$$

Recall,

$$\int x^2 \sin(x)\,dx = -x^2\cos(x) + 2\int x\cos(x)\,dx$$

Substitute (3) into (2)

$$\int x^2 \sin(x)\,dx = -x^2\cos(x) + 2(x\sin(x) + \cos(x))$$

$$\int x^2 \sin(x)\,dx = -x^2\cos(x) + 2x\sin(x) + 2\cos(x) + C$$

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2. $\int 3te^{2t}\, dt$

$\int u\, dv = uv - \int v\, du$

let $u = 3t$

$\frac{du}{dt} = 3 \qquad du = 3dt$

let $dv = e^{2t}\, dt$

$v = \frac{1}{2} e^{2t}$

$\int 3t(e^{2t})\, dt = 3t(1/2)\, e^{2t} - \int 1/2\, e^{2t} \cdot 3dt$

$= \frac{3}{2} te^{2t} - \frac{3}{4} e^{2t}$

3 $\int 2x^2 \times \ln x\, dx$

$2 \times \int x^2 \ln(x)\, dx$

$2 \times \ln(x) * x^2\, dx$

$\int u dv = uv - \int v\, du$

$u = \ln(x)$

$dv = x^2 dx$

$v = \int dv \qquad du = \frac{1}{x} dx \qquad v = \frac{x^3}{3}$

$2\left(\ln(x) \times \frac{x^3}{3} - \int \frac{x^3}{3} \times \frac{1}{x}\, dx\right)$

$\int u dv = uv - \int v\, du$

$\int 2x^2 \ln x\, dx = 2(\ln x \times \frac{x^3}{3}) - \int \frac{x^3}{3} \times \frac{1}{x}\, dx$

$2x^2 \ln x\, dx = 2(\ln x \times x^3/3 - \int x^2/3\, dx$

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$2(\ln x \times \frac{x^3}{3} - \frac{1}{3} x \int x^2\,dx)$

$\int x^2\,dx = \frac{x^{n+1}}{n+1}$

$\int x^2\,dx = \frac{x^{2+1}}{2+1} = \frac{x^3}{3}$

$= 2(\ln(x) \times \frac{x^3}{3} - \frac{1}{3} \times \frac{x^3}{3})$

$= \frac{2x^3 \times \ln(x)}{3} - \frac{2x^3}{9}$

$\int 2x^2 \ln x\,dx = \frac{2x^3 \times \ln x}{3} - \frac{2x^3}{9} + c$

4 $\int \frac{2x - 3x^2}{1-x}\,dx$, Seperate $= \int \frac{2x}{1-x} - \frac{3x^2}{1-x}\,dx$

$\int f(x) \pm g(x)\,dx = \int f(x)\,dx \pm \int g(x)\,dx$

$\int \frac{2x}{1-x}\,dx - \int \frac{3x^2}{1-x}\,dx$

Calculate undefinite integral

$2 - 2x - 2\ln(|1-x|) + \frac{\cancel{9} + 6x + 3}{2} + 3\ln(|1-x|)$

$\therefore \frac{-5 + 2x + 3x^2}{2} + \ln(|1-x|)$

$= \int \frac{2x - 3x^2}{1-x} = \frac{-5 + 2x + 3x^2}{2} + \ln(|1-x|)$