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Question:
1. Find the equation of the tangent at the point (1,0) on the circle $x^2+y^2-5x-y+4=0$

Solution

$x^2+y^2-5x-y+4=0$ to be equation (i)

equation of a line $= y-y_1 = m(x-x_1)$

Re-arranging equation (i)

$x^2-5x+y^2+y+4=0$. by factorizing, we have;

$\left(x-\frac{5}{2}\right)^2-\frac{25}{4}+\left(y-\frac{1}{2}\right)^2-\frac{1}{4}+4=0$

Collect like terms

$\left(x-\frac{5}{2}\right)^2+\left(y-\frac{1}{2}\right)^2-\frac{5}{2}=0$

$\left(x-\frac{5}{2}\right)^2+\left(y-\frac{1}{2}\right)^2=\frac{5}{2}$

∴ The centre is $\left(\frac{5}{2},\frac{1}{2}\right)$ and radius is $=\sqrt{\frac{5}{2}}$

Let P = point (1,0) and C = centre $\left(\frac{5}{2},\frac{1}{2}\right)$.

Gradient of $CP = \frac{\Delta y}{\Delta x} = \frac{0-\frac{1}{2}}{1-\frac{5}{2}} = \frac{-\frac{1}{2}}{-\frac{3}{2}}$

Gradient (m) of $CP = -\frac{1}{2} \times -\frac{2}{3} = -5$

Since they are perpendicular, i.e at 90°, Gradient of the tangent $= \frac{1}{5}$

Equation of the tangent therefore will be:

$y-y_1 = m(x-x_1)$

$y-0 = \frac{1}{5}(x-1)$

$\therefore y = \frac{x}{5}+\frac{1}{5}$ //

2. find the equation of the tangent at the point (1,0) on the circle $x^2+y^2-12x-12y+47=0$

Solution

$x^2+y^2-12x-12y+47=0$ to be equation (1)

Let equation of the line $= y-y_1 = m(x-x_1)$

re-arranging equation (1) and let P = point (1,0)

$x^2-12x+y^2-12y+47=0$

factorizing.

$(x-6)^2-36+(y-6)^2-36+47=0$, Collect like terms

$(x-6)^2+(y-6)^2-25=0$

$(x-6)^2+(y-6)^2=25$

Centre (c) is (6, 6) and radius $r=\sqrt{25}=5$.

Gradient of $(P = \frac{\Delta y}{\Delta x} = \frac{y_2-y_1}{x_2-x_1} = \frac{0-6}{1-6} = \frac{+6}{+5} = \frac{6}{5}$

They are perpendicular

Gradient of Tangent $= -5/6$

equation of Tangent $P = y-y_1 = m(x-x_1)$

$y-0 = -\frac{5}{6}(x-1)$,

$y = -5/6x + 5/6$

3) find the equation of the tangent at the point (1,0) on the circle $x^2+y^2-8x+4y+40=0$

Solution

$x^2+y^2-8x+4y+40=0$ to be equation (1)

Equation of a line $= y-y_1 = m(x-x_1)$.

For centre = C, re-arranging equation (1) we have

$x^2+y^2-8x+4y+40=0$

$x^2-8x+y^2+4y+40=0$

$(x-4)^2-16+(y+2)^2-4+40=0$

$(x-4)^2 + (y+2)^2 + 20 = 0$

$(x-4)^2 + (y+2)^2 = -20$

∴ Centre $(c) = (4, -2)$, Radius $= \sqrt{20}$

Gradient of $CP = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0+2}{1-4} = \frac{2}{-3} = -\frac{2}{3}$

Since they are at 90°

The gradient of the target $= 3/2$

Therefore, the equation of Target:

$y - y_1 = m(x - x_1)$

$y - 0 = \frac{3}{2}(x - 1)$

$y = \frac{3x}{2} - \frac{3}{2}$ //