Egwuatu Dugidem Ifeduwa 19/sci01/042

i) $x^2 \sin x\, dx$

$\int x^2 \sin x\, dx$

Using: $\int u\,dv = uv - \int v\,du$

for $\int u\,dv = \int x^2 \sin x\, dx$ we let:

$u = x^2,\ \frac{du}{dx} = 2x \Rightarrow du = 2x\,dx$

$dv = \sin(x)\,dx \Rightarrow \int dv = \int \sin(x)\,dx \Rightarrow v = -\cos x$

$= \int x^2 \sin x\, dx = -x^2 \cos x - \int(-2x \cos x\, dx)$

$= \int x^2 \sin(x)\,dx = -x^2 \cos x + 2\int x \cos x\, dx$

$u = x \Rightarrow \frac{du}{dx} = 1 \Rightarrow du = dx$

$dv = \cos x\, dx \Rightarrow \int dv = \int \cos x\, dx \Rightarrow v = \sin x$

$\therefore \int x \cos x\, dx = x \sin x - \int \sin x\, dx$

Since $\int \sin x\, dx = -\cos x \Rightarrow \int x \cos x\, dx = x \sin x + \cos x$

Going back $= \int x^2 \sin x\, dx = x^2 \cos x + 2\int x \cos x\, dx$

Substitute $\int x\cos x\,dx = x\sin x + \cos x$

$\int x^2 \sin x\,dx = -x^2\cos(x) + 2x\sin x + \cos x$

$\therefore \int x^2 \sin(x)\,dx = -x^2\cos x + 2x\sin x + 2\cos x + C$

2) ~~[illegible]~~ First we do $te^{2t}dt$ then multiply by 3 later

by parts $\therefore u = t$

$du = dt$

$dv = e^\wedge(2t)dt$

$v = (1/2)e^\wedge(2t)$

$Uv = (1/2)te^\wedge(2t)$

$vdu = (1/2)e^\wedge(2t)$

Uv - integral $vdu = (1/2)te^{2t} - e^{2t}$

integral $(u\,dv) = Uv$ - integral $(v\,du)$

Let $u = 3t$, $du/dt = 3$, so $du = 3dt$.

let $dv = e^\wedge(2t)dt$

$v = (1/2)e^{2t}$

So integral $(3t(e^\wedge(2t))dt = 3t(1/2)e^{2t}$ - integral $1/2\,e^{2t}3dt$

$= (3/2)te^{2t} - (3/4)e^{2t}$

let $3t = u$ and $e^{2t}dt = dv$

$du = 3dt$ $v = (1/2)e^{2t}$

The integral is Uv - integral vdu

$= (3/2)t\,e^{2t}$ - integral $(3/2)e^{2t}$

$= (3/2)te^{2t} - (3/4)e^{2t}$

3) $\int 2x^2 \times \ln x \, dx$
(Commutative property)

$2 \times \int x^2 \ln(x) dx$

$2 \times \ln(x) \times x^2 dx$

$\int v\,du = uv - \int u\,du$

$u = \ln(x) \quad dv = x^2 dx$

$v = \int dv$

$du = 1/x \, dx \quad v = x^3/3$

$2\left(\ln(x) \times x^3/3 - \int x^3/3 \times 1/x \, dx\right)$

$\int u\,dv = uv - \int v\,du$

$\int 2x^2 \ln x \, dx = 2(\ln x \times x^3/3) - \int x^3/3 \times \frac{1}{x} dx$

$2x^2 \ln x \, dx = 2(\ln x \times x^3/3 - \int x^2/3 \, dx \, dx$

$= 2(\ln x \times \frac{x^3}{3} - 1/3 \times \int x^2 dx$

$\int x^n dx = \frac{x^{n+1}}{n+1}$ $= \int x^2 dx = \frac{x^{2+1}}{2+1}$ $= \frac{x^3}{3}$

$2\left(\ln(x) \times \frac{x^3}{3} - \frac{1}{3} \times x^3/3\right)$

$= \frac{2x^3 \times \ln(x)}{3} - \frac{2x^3}{9}$

$\int 2x^2 \ln x \, dx = \frac{2x^3 \times \ln x}{3} - \frac{2x^3}{9} + 1$

4) $\int \frac{2x - 3x^2}{1-x} dx$, separate $= \int \frac{2x}{1-x} - \frac{3x^2}{1-x} dx$

Property of Integral

$\int f(x) \pm g(x) dx = \int f(x) dx \pm \int g(x) dx$ $= \int \frac{2x}{1-x} dx - \int \frac{3x^2}{1-x} dx$

Calculate Indefinite integral

$2 - 2x - 2\ln(|1-x|) + \frac{-9 + 6x + 3x^2}{2} + 3\ln(|1-x|)$

Simplify: $\frac{-5 + 2x + 3x^2}{2} + \ln(|1-x|)$

$\therefore \int \frac{2x - 3x^2}{1-x} = \frac{-5 + 2x + 3x^2}{2} + \ln(|1-x|)$