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1. $\int \frac{2x}{\sqrt{4x^2-1}}\,dx$

solution

Let $u = 4x^2 - 1$

$\left(\frac{u-1}{4}\right)^{1/2} = x$

$x = \left(\frac{u-1}{4}\right)^{1/2} \Rightarrow \frac{(u-1)^{1/2}}{2}$

$dx/du = \frac{1}{4(u-1)^{1/2}}$

$dx = du/4(u-1)^{1/2}$

$\int \frac{2x}{\sqrt{4x^2-1}}\,dx = \int \frac{2(u-1)^{1/2}}{2} \cdot \frac{1}{u^{1/2}} \cdot \frac{du}{4(u-1)^{1/2}}$

$= \frac{1}{4}\int u^{-1/2}\,du$

$= \frac{1}{4}\left[\frac{u^{1/2}}{1/2}\right] + C$

$= \cancel{\frac{2}{4}}\frac{1}{2}\left[(4x^2-1)^{1/2}\right] + C$

$= \int \frac{2x}{\sqrt{4x^2-1}}\,dx = \frac{1}{2}\left[\sqrt{4x^2-1}\right] + C$

2. $\int \frac{\sin^{-1}x}{\sqrt{1-x^2}}\,dx$

Let $u = \sin^{-1}x$

$\frac{du}{dx} = \frac{1}{\sqrt{1-x^2}}$

$dx = (\sqrt{1-x^2})\,du$

$\int \frac{\sin^{-1}x}{\sqrt{1-x^2}}\,dx = \int \frac{u}{\sqrt{1-x^2}} \times \sqrt{1-x^2}\,du$

$= \int \frac{\sin^{-1}x}{\sqrt{1-x^2}}\,dx = \int u\,du$

$\int \frac{\sin^{-1}x}{\sqrt{1-x^2}}\,dx = \frac{u^2}{2} + C$

$$\int \frac{\sin^{-1}x}{\sqrt{1-x^2}}\,dx = \frac{(\sin^{-1}x)^2}{2} + c$$

3. $\int (\tan x)^6 \sec^2 x\, dx$

$u = \tan x, \quad du/dx = \sec^2 x\, dx$

$dx = du / \sec^2 x$

$= \int u^6 \cdot \sec^2/x \cdot du/\sec^2 x$

$= u^7/7 + C$

$$\int (\tan x)^6 \sec^2 x\, dx = \frac{(\tan x)^7}{7} + C$$