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MATRIC NO: 18/SCI01/013

QUESTIONS

- 1). Explain with 2 examples what you understand by linear transformation.
- 2). Given a linear transformation of a matrix operator A on a vector x, compute T(x) if $A = \begin{pmatrix} 1 & 9 & 3 \\ -2 & 6 & 7 \\ 0 & -1 & 3 \end{pmatrix}$ and

$$x = \begin{pmatrix} 1 \\ 4 \\ -8 \end{pmatrix}$$

$$4$$

$$-8$$

- 3). Define completely with mathematical examples what you understand by Rank of a matrix.

SOLUTION

1. A linear transformation is a function from one vector space to another that respects the underlying (linear) structure of each vector space. It is also known as a linear operator or map.

Examples

- (a.) The **expected value** of a random variable (which is in fact a function, and a member of a vector space)

$$E(X + Y) = E(X) + E(Y) \text{ and } E[aX] = aE[X].$$

- (b.) An identity map on any module is a linear transformation.

$$2. \begin{pmatrix} 1 & 9 & 3 \\ -2 & 6 & 7 \\ 0 & -1 & 3 \end{pmatrix} X = \begin{pmatrix} 1 \\ 4 \\ -8 \end{pmatrix}$$

$$T(x) = A(x)$$

$$T(x) = 1 \begin{pmatrix} 1 \\ -2 \\ 0 \end{pmatrix} + 4 \begin{pmatrix} 9 \\ 6 \\ -1 \end{pmatrix} - 8 \begin{pmatrix} 3 \\ 7 \\ 3 \end{pmatrix}$$

$$\begin{aligned}
& \begin{pmatrix} 1 \\ -2 \\ 0 \end{pmatrix} + \begin{pmatrix} 36 \\ 24 \\ -4 \end{pmatrix} + \begin{pmatrix} -24 \\ -56 \\ -24 \end{pmatrix} \\
&= \begin{pmatrix} 13 \\ -34 \\ -28 \end{pmatrix} \quad \text{Hence, the transformation of} \quad \begin{pmatrix} 1 \\ 4 \\ -8 \end{pmatrix}
\end{aligned}$$

3. The rank of a matrix is defined as the maximum number of linearly independent column vector in the matrix or maximum number of linearly independent row vector in the matrix.

Example

$$\mathbf{A} = \begin{pmatrix} 1 & -3 & 6 \\ 4 & 0 & 2 \\ 8 & 5 & 1 \end{pmatrix}$$

(i) **RANK OF A**

$$|\mathbf{A}| = \begin{vmatrix} 1 & -3 & 6 \\ 4 & 0 & 2 \\ 8 & 5 & 1 \end{vmatrix}$$

$$= 1 \begin{vmatrix} 0 & 2 \\ 5 & 1 \end{vmatrix} - (-3) \begin{vmatrix} 4 & 2 \\ 8 & 1 \end{vmatrix} + 6 \begin{vmatrix} 4 & 0 \\ 8 & 5 \end{vmatrix}$$

$$= 1(0 - 10) + 3(4 - 16) + 6(20 - 0)$$

$$= -10 - 36 + 120$$

$$= 74$$

Since $|\mathbf{A}| \neq 0$, The Rank of A is 3

Note: But if the determinant of the matrix is equal to 0 then we delete row and column of the 3 x 3 matrix forming a 2 x 2 matrix then we find the determinant of the matrix. If we get an answer not equal to 0 then the matrix of rank 2

