$$\Delta y = \frac{4x^3 - 4(\Delta x + x)^3}{x^3(\Delta x + x)^3}$$

$$\Delta y = \frac{4x^3 - 4\left[x^3 + 3x^2(\Delta x) + 3x(\Delta x)^2 + (\Delta x)^3\right]}{x^3\left[x^3 + 3x^2(\Delta x) + 3x(\Delta x)^2 + (\Delta x)^3\right]}$$

$$\Delta y = \frac{4x^3 - 4x^3 - 12x^2(\Delta x) - 12x(\Delta x)^2 - 4(\Delta x)^3}{x^3(\Delta x + x)^3}$$

$$\frac{\Delta y}{\Delta x} = \frac{-12(\Delta x)}{x^3(\Delta x + x)^3} \times \frac{1}{\Delta x} - \frac{12x^2(\Delta x)^2}{x^3(\Delta x + x)^3} \times \frac{1}{\Delta x} - \frac{4(\Delta x)^3}{x^3(\Delta x + x)^3} \times \frac{1}{\Delta x}$$

$$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \frac{-12x^2}{x^3(0+x)^3} - 0 - 0$$

$$\frac{dy}{dx} = \frac{-12x^2}{x^6}$$

$$\frac{dy}{dx} = -12x^{-4}$$

Number 2) $\int \frac{1}{x^2+36}\, dx$

$u = x/6 \rightarrow \frac{dy}{dx} = \frac{1}{6}$

$dx = 6\,dy$

$\int \frac{6}{36u^2+36}\, du$

$= \frac{1}{6}\int \frac{1}{u^2+1}\, du$

also $\int \frac{1}{u^2+1}\, du = \arctan(u)$

$\frac{1}{6}\int \frac{1}{u^2+1}\, du = \frac{\arctan(u)}{6}$

recall $u = \frac{x}{6}$

$\therefore \int \frac{1}{x^2+36}\, dx = \frac{\arctan\left(\frac{x}{6}\right)}{6} + c$

b) $\int \frac{1}{x^2+13}\, dx$

$u = \frac{x}{\sqrt{13}} \longrightarrow \frac{du}{dx} = \frac{1}{\sqrt{13}}$

$dx = \sqrt{13}\, du$

$\int \frac{\sqrt{13}}{3u^2+13}\, du$

$= \frac{1}{\sqrt{13}}\int \frac{1}{u^2+1}\, du$

also $\int \frac{1}{u^2+1}\, du = \arctan(u)$

$\frac{1}{\sqrt{13}}\int \frac{1}{u^2+1}\, du = \frac{\arctan(u)}{\sqrt{13}}$

recall $u = \frac{x}{\sqrt{13}}$

$\therefore \int \frac{1}{x^2+13}\, dx = \frac{\arctan\left(\frac{x}{\sqrt{13}}\right) + c}{\sqrt{13}}$

NAME : A MBA MATHEMATICS
COLLEGE : SCIENCES
COURSE : MAT 104
MATRIC NUMBER : 19/sc109/004

1) find the derivative of the following using first Principle
$y = \sin(5/x^2)$ (b) $y = 4/x^3$
find the integral of the following
a) $dx/(6x^2+3x)$ b) $dx/(x^2+13)$

Solution.

(a) $y = \sin(3/x^2)$

$y = \sin 3/x^2$

$y = \sin 3x^{-2}$

let $u = 3x^{-2}$

$y = \sin u$

$\Delta y + y = \sin(u + \Delta u)$

$\Delta y = \sin(u+\Delta u) - y$

$\Delta y = \sin(u+\Delta u) - \sin u$

recall $\sin A - \sin B = 2\cos\frac{A+B}{2}\sin\frac{A-B}{2}$

$A = u+\Delta u \quad B = u$

$\frac{A+B}{2} = \frac{u+\Delta u+u}{2} = \frac{2u+\Delta u}{2} = u + \frac{\Delta u}{2}$

$\frac{A-B}{2} = \frac{u+\Delta u-u}{2} = \frac{\Delta u}{2}$

Hence $\frac{\Delta y}{\Delta u} = \frac{2\cos\left(u+\frac{\Delta u}{2}\right)\sin\left(\frac{\Delta u}{2}\right)}{\Delta u}$

Or

$\frac{\Delta y}{\Delta u} = \frac{\cos\left(u+\frac{\Delta u}{2}\right)\sin\left(\frac{\Delta u}{2}\right)\times 1/2}{\Delta u \times 1/2}$

$\frac{\Delta y}{\Delta u} = \text{Since } \lim_{x\to 0} = 1$

$\frac{\Delta y}{\Delta u} = \cos(u+0) \lim_{\Delta x\to 0} \left(\frac{\Delta u}{2}\right)/\frac{\Delta u}{2}$

$\frac{dy}{du} = \cos u \times 1$

$\frac{dy}{du} = \cos u$