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Q1 Integrate the following with respect to their variable

1. $3te^{2t}$

Solution

$$\int 3te^{2t}\,dt$$

$$3\int te^{2t}\,dt$$

recall:

$$\int u\,dv = uv - \int v\,du$$

where

$u = t$, $dv = e^{2t}$

$du = dt$, $\int dv = \int e^{2t}$

$v = \frac{1}{2}e^{2t}$

Substituting,

$t \cdot \frac{1}{2} e^{2t} - \int \frac{1}{2} e^{2t} \, dt$

$= \frac{1}{2} te^{2t} - \frac{1}{2} \int e^{2t} \cdot dt$

$= \frac{1}{2} te^{2t} - \frac{1}{4} e^{2t} + C$

$3\left( \frac{1}{2} te^{2t} - \frac{1}{4} e^{2t} + C \right)$

$= \frac{3}{2} te^{2t} - \frac{3e^{2t}}{2} + C$

ans.

(2) $x^2 \sin x$

solution

$\int x^2 \sin x \, dx = uv - \int v \, du$

where; $u = x^2$, $du = 2x \, dx$, $dv = \sin x$

$v = -\cos x$

$= -x^2 \cos x - \int -\cos x \, 2x \, dx$

$= -x^2 \cos x + \int 2x \cos x \, dx$

$= -x^2 \cos x + 2x \sin x + \cos x + C$

ans.

3) $\sin 7x \cos x$

Solution

$\int \sin 7x, \cos 2x$

Recall;

$\sin A \cdot \cos B = \frac{1}{2}[\sin(a+b) + \sin(a-b)]$

$\therefore \sin(7x)\cos(2x) = \frac{1}{2}[\sin(9x) + \sin(5x)]$

$= \frac{1}{2}\int \sin(9x)\,dx + \int \frac{1}{2}\sin(5x)\,dx$

$= \frac{1}{2}\left(\frac{-\cos(9x)}{9}\right) + \frac{1}{2}\left(\frac{-(\cos(5x))}{5}\right) + C$

$= \frac{-\cos(9x)}{18} - \frac{\cos(5x)}{10} + C$

ans.

H) $\frac{(2x - 3x^2)}{1-x}$

solution

$\int \frac{2x - 3x^2}{1-x}\,dx$

Problem Evaluation

let $u = 2x - 3x^2$

$du = 2 - 6x\,dx$

$dx = \frac{du}{2-6x}$

$$\int \frac{u}{(1-x)(2-6x)}\,du = \int \frac{A}{(1-x)} + \frac{B}{(2-6x)}$$

multiply through by $(1-x)(2-6x)$

$2x - 3x^2 = (2-6x)A + B(1-x)$

when $x = 1$, when $x = 1/3$

$-1 = -4A$ $\quad 1/3 = 2/3\,B$

$A = 1/4$ $\quad B = 1/2$

$\int 4(1-x) + \int 2(2-6x)$

$-4/2\,(x) + -2/2\,(x) + C$

$= -2x - x + C$

$= -3x + C$

$\Longrightarrow$ ans.